Gradual C0: Symbolic Execution for Gradual Verification

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Current static verification techniques such as separation logic support a wide range of programs. However, such techniques only support complete and detailed specifications, which places an undue burden on users. To solve this problem, prior work proposed gradual verification, which handles complete, partial, or missing specifications by soundly combining static and dynamic checking. Gradual verification has also been extended to programs that manipulate recursive, mutable data structures on the heap. Unfortunately, this extension does not reward users with decreased dynamic checking as specifications are refined. In fact, all properties are checked dynamically regardless of any static guarantees. Additionally, no full-fledged implementation of gradual verification exists so far, which prevents studying its performance and applicability in practice.

We present Gradual C0, the first practicable gradual verifier for recursive heap data structures, which targets C0, a safe subset of C designed for education. Static verifiers supporting separation logic or implicit dynamic frames use symbolic execution for reasoning; so Gradual C0, which extends one such verifier, adopts symbolic execution at its core instead of the weakest liberal precondition approach used in prior work. Our approach addresses technical challenges related to symbolic execution with imprecise specifications, heap ownership, and branching in both program statements and specification formulas. We also deal with challenges related to minimizing insertion of dynamic checks and extensibility to other programming languages beyond C0. Finally, we provide the first empirical performance evaluation of a gradual verifier, and found that on average, Gradual C0 decreases run-time overhead between 11-34% compared to the fully-dynamic approach used in prior work. Further, the worst-case scenarios for performance are predictable and avoidable. This work paves the way towards evaluating gradual verification at scale.

CCS Concepts:
- Theory of computation → Logic and verification; Automated reasoning; Hoare logic; Separation logic.

1 INTRODUCTION

Separation logic [Reynolds 2002] supports the modular static verification of heap-manipulating programs. Its variant Implicit dynamic frames (IDF) [Smans et al. 2009] and extension with recursive...

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abstract predicates [Parkinson and Bierman 2005; Smans et al. 2009] further support verifying recursive heap data structures, such as trees, lists, and graphs. While these techniques allow users to specify and verify more code than ever before, tools implementing them (e.g., Viper [Müller et al. 2016], VeriFast [Jacobs et al. 2011], Chalice [Leino et al. 2009], JStar [Distefano and Parkinson J 2008], and SmallFoot [Berdine et al. 2006]) are still largely unused due to the burden they place on their users. Such tools poorly support partial specifications, and thus require users to provide a number of auxiliary specifications (such as folds, unfolds, loop invariants, and inductive lemmas) in an all or nothing fashion to support inductive proofs of correctness. The tools also require many of these auxiliary specifications to be written before they can provide feedback on the correctness of specifications for important functional properties. For example, to prove that a simple insertion function preserves list acyclicity, static verifiers need 1.5 times as many lines of auxiliary specifications to program code (§2). They also need a significant number of these auxiliary specifications to uncover problems with specifications of the acyclic property (§2).

Inspired by gradual typing [Siek and Taha 2006], Bader et al. [2018] proposed gradual verification to support the incremental specification and verification of software. Users can write imprecise (i.e., partial) specifications backed by run-time checking where necessary. An imprecise formula can be fully unknown, written ?, or combine a static part with the unknown, as in ? ∗ x. f == 2. Wise et al. [2020] extend Bader et al. [2018]’s initial system by designing and formalizing the first gradual verifier for recursive heap data structures. It supports imprecise specifications with accessibility predicates from IDF and abstract predicates; and thus, also (in theory) the run-time verification of these constructs. During static verification, an imprecise specification can be optimistically strengthened (in non-contradictory ways) by the verifier to support proof goals. Wherever such strengthenings occur, dynamic checks are inserted to preserve soundness. Gradual verification smoothly supports the spectrum between static and dynamic verification. This is captured by properties adapted from gradual typing [Siek et al. 2015], namely the gradual guarantee, stating that the verifier will not flag static or dynamic errors for specifications that are correct but imprecise, and the fact that gradual verification conservatively extends static verification, i.e., they coincide on fully-precise programs.

While promising, Wise et al. [2020]’s gradual verifier has neither been implemented nor validated in practice. Furthermore, their design relies on weakest liberal preconditions [Dijkstra 1975] for static reasoning rather than symbolic execution [King 1976], which is the ideal reasoning technique for tools based on separation logic or IDF. Indeed, Viper [Müller et al. 2016], VeriFast [Jacobs et al. 2011], JStar [Distefano and Parkinson J 2008], and SmallFoot [Berdine et al. 2006] all support these permission logics with symbolic execution, not weakest liberal preconditions. Finally, Wise et al. [2020]’s gradual verifier is not efficient in the sense that it checks all memory safety and functional properties dynamically regardless of the precision of specifications.

This paper presents the design, implementation, and validation of Gradual C0—1—the first gradual verifier for imperative programs manipulating recursive heap data structures that is based on symbolic execution. Gradual C0 targets C0, a safe subset of C designed for education, with appropriate support (and pedagogical material) for dynamic verification. Technically, Gradual C0 is built on top of the Viper static verification infrastructure [Müller et al. 2016], which facilitates the development of program verifiers supporting IDF and recursive abstract predicates. Gradual C0’s back-end leverages this infrastructure to simplify the implementation of gradual verifiers for other programming languages, and Gradual C0’s front-end demonstrates how this is done for C0. Further, Gradual C0 minimizes the insertion of dynamic checks using statically available information and optimizes the checks’ overhead at run time.

1Gradual C0 is hosted on Github: https://github.com/gradual-verification/gvc0.
Overall, we address new technical challenges in gradual verification related to symbolic execution, extensibility to multiple programming languages, and minimizing run-time checks and their overhead:

- Gradual C0’s symbolic execution algorithm is responsible for statically verifying programs with imprecise specifications and producing minimized run-time checks. In particular, Gradual C0 tracks the branch conditions created by program statements and specifications to produce run-time checks for corresponding execution paths. At run time, branch conditions are assigned to variables at the branch point that introduced them, which are then used to coordinate the successive checks as required. Further, Gradual C0 creates run-time checks by translating symbolic expressions into specifications—reversing the symbolic execution process.

- The run-time checks produced by Gradual C0 contain branch conditions, simple logical expressions, accessibility predicates, separating conjunctions, and predicates. Each of these constructs is specially translated into source code that can be executed at run time for dynamic verification. Logical expressions are turned into assertions. Accessibility predicates and separating conjunctions are checked by tracking and updating a set of owned heap locations. Finally, predicates are translated into recursive boolean functions. By encoding run-time checks into C0 source code, we avoid complexities from augmenting the C0 compiler to support dynamic verification. We also design these encodings to be performance friendly, e.g. owned heap locations are tracked in a dynamic hash table.

Work on gradual typing performance has shown that minimizing the insertion of dynamic checks does not trivially correlate with overall execution performance; the nature of the inserted checks (such as higher-order function wrappers) as well as their location in the overall execution flow of a program can have drastic and hard-to-predict consequences [Campora et al. 2018; Feltey et al. 2018; Takikawa et al. 2016]. Therefore, our validation of Gradual C0 aims to empirically evaluate the relationship between minimizing check insertion and observed run-time performance in gradual verification. We evaluate the performance of Gradual C0 by adapting Takikawa et al. [2016]’s performance lattice method to gradual verification, exploring the performance characteristics for partial specifications of four common data structures. This method models the gradual verification process as a series of steps of partial specifications from an unspecified program (containing all ?s) to a statically verifiable specification (not containing any ?s) where, at each step, an atomic conjunct is added to the current, partial specification. Statically, we observe that as more specifications are added, more verification conditions can be statically discharged. Though imprecision introduces unavoidable run-time checks, gradual verification decreases run-time overhead by an average of 11-34% compared to dynamic verification (and thus Wise et al. [2020]’s approach). Sources of run-time overhead correspond to the predictions made in prior work, and our study shows that the gradual guarantee holds empirically for our tool across thousands of sampled imprecise specifications.

2 GRADUAL C0 IMPROVES THE STATIC SPECIFICATION PROCESS

Static verification tools, like Viper [Müller et al. 2016], VeriFast [Jacobs et al. 2011], Chalice [Leino et al. 2009], JStar [Distefano and Parkinson J 2008], and SmallFoot [Berdine et al. 2006], require a number of user-provided auxiliary specifications, such as folds, unfolds, lemmas, and loop invariants, to prove properties about recursive heap data structures. Worse, they also require users to write many of these auxiliary specifications before the tools can provide useful feedback on the correctness of other specifications, including ones containing important functional properties. Therefore, users are burdened by writing many detailed and extraneous specifications with inadequate static feedback through the process. In this section, we illustrate this burden with a simple list insertion example.
(inspired by a similar introductory example and discussion from Wise et al. [2020]) and output from Viper. Then, we show in §3 how Gradual C0 overcomes this burden by smoothly supporting the spectrum between static and dynamic checking. Users can avoid writing auxiliary specifications and still get sound verification of their code with increased run-time checking. Users can also receive run-time feedback on the correctness of their specifications very early in the specification process, and the resulting error messages closely align with inherent problems in the specifications or in the program, making debugging them easier.

Fig. 1 implements a linked list and function that inserts a new node at the end of a given list, called insertLast, in C0 [Arnold 2010]. The insertLast function traverses the list to its end with a while loop starting from the root. That is, insertLast implicitly assumes the list is non-empty (non-null) and acyclic; and that for multiple successive calls to insertLast the list remains acyclic and non-empty after insertion. These facts can be proven explicitly with static verification; the complete static specification is given in Fig. 2, highlighted in grey.

List acyclicity is specified with two predicates acyclicSeg and acyclic:

```
predicate acyclicSeg(Node* s, Node* e) =
    s == e ? true : acc(s->val) & acc(s->next) & acyclicSeg(s->next, e)
predicate acyclic(Node* n) = acyclicSeg(n, NULL)
```

The acyclicSeg predicate uses accessibility predicates and the separating conjunction from IDF. Ownership is ensured through accessibility predicates such as `acc(s->val)`; and, `acc(s->val) && s->val == 2` states that `s->val` is uniquely owned and contains the value 2. The separating conjunction, denoted by `&&`, ensures memory disjointness: `acc(s->next) && acc(s->next->next)` states that the heap locations `s->next` and `s->next->next` are distinct (i.e. `s != s->next`) and are each owned.

Thus, the abstract recursive predicate acyclicSeg, which can be thought of as a pure boolean function, specifies that a list segment is acyclic. That is, `acyclicSeg(s, e)` denotes that all heap locations in list `s` are distinct up to node `e` by recursively generating accessibility predicates for each node in `s` up to `e`, joined with the separating conjunction. Further, `acyclic(n, NULL)` denotes that all heap locations in list `n` are distinct and so `n` is acyclic, as specified with `acyclic(n)`.

Now that we have specified `acyclic`, we use it in `insertLast`’s precondition (line 5) and postcondition (lines 6-7) to denote preservation of list acyclicity. We also specify that `insertLast` preserves list non-nullness with simple comparison logic (i.e. `list != NULL` and `\result != NULL`). Ideally, we would stop here and static verifiers would be able to prove `insertLast`’s implementation is correct with respect to this specification; however, as you can see in Fig. 2 such tools require many more specifications. In fact, there are 27 lines of auxiliary specifications (comprised of folds, unfolds, loop invariants, and inductive lemmas); in contrast to 18 lines of wanted specifications (the predicates and pre- and postconditions) and program code. Furthermore, these auxiliary specifications are complex, as discussed next.

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2Most IDF papers use `*` to denote the separating conjunction. We follow Viper and use `&&` instead.
/*@ predicate acyclicSeg(Node* s, Node* e) =
(s == e) ? true : acc(s->val) && acc(s->next) && acyclicSeg(s->next, e); @*/

//@ predicate acyclic(Node* n) = acyclicSeg(n, NULL);

Node * insertLast(Node * list, int val)
//@ requires acyclic(list) && list != NULL ;
//@ ensures acyclic(
result) &&
result != NULL ; @*/
{//@ unfold acyclic(list);
//@ unfold acyclicSeg(list, NULL);
Node * y = list ;
//@ fold acyclicSeg(list, y);
while (y->next != NULL )
//@ loop_invariant
acyclicSeg(list, y) &&
acc(y->next) && acc(y->val) &&
acyclicSeg(y->next, NULL) ; @*/
{//@ unfold acyclicSeg(y, NULL);
//@ fold acyclicSeg(tmp->next, y);
//@ fold acyclicSeg(tmp, y);
mergeLemma(list, tmp, y);
}
//@ unfold acyclicSeg(y->next->next, NULL);
//@ fold acyclicSeg(y->next, NULL);
//@ fold acyclicSeg(y, NULL);
mergeLemma(y, NULL);
//@ fold acyclic(list):
return list ;
}

void mergeLemma (Node* a, Node* b, Node* c)
//@ requires acyclicSeg(a, b) && acyclicSeg(b, c);
//@ ensures acyclicSeg(a, c);
{ if (a == b) {
} else {
//@ unfold acyclicSeg(a, b);
mergeLemma(a->next, b, c);
//@ fold acyclicSeg(a, c);
} }

2.1 Auxiliary Specifications

Static verifiers cannot reliably unroll recursive predicates during verification; so, such tools rely on explicit unfold and unfold statements to control the availability of predicate information in the verifier. This treats predicates iso-recursively; while an equi-recursive interpretation treats predicates as their complete unrolling [Summers and Drossopoulou 2013]. Consequently, the acyclic and acyclicSeg predicates are unfolded and folded often in Fig. 2 (lines 9-10, 12, 20-22, 30-32, and 34). Looking closely, we see that acyclic(list), which is assumed true from the precondition, is unfolded on line 9. This consumes acyclic(list) and produces its body acyclicSeg(list, NULL), which is subsequently unfolded on line 10. Then, at the fold on line 34, the body of acyclic(list) is packed up into the predicate itself to prove the list remains acyclic after insertion.

Additionally, static verifiers cannot tell if or when a loop will end (in our example the verifier cannot tell when the list being iterated over ends), but must verify all paths through the program. Therefore, static verifiers reason about loops using specifications called loop invariants, which are properties that are preserved for each execution of the loop including at entry and exit. Further, loop invariants must also provide information necessary for proof obligations after the loop, e.g. that the list in insertLast is acyclic after insertion. In Fig. 2, these constraints result in the loop invariant on lines 14-16 that segments the list into three disjoint and acyclic parts: from the root up to the current node y (acyclicSeg(list, y)), the current node y (acc(y->val) && acc(y->next)), and from the node after y to the end (acyclicSeg(y->next, NULL)). Exposing y via its accessibility predicates provides access to y->next on line 19 in the loop body; and, acyclicSeg(list, y) helps prove acyclic(list) holds after the loop, as we will see next.
Node* insertLast(Node* list, int val)
//@ requires acyclic(list) && list != NULL ;
//@ ensures acyclic(result) &&
\result != NULL ; @*/
{
//@ unfold acyclic(list);
//@ unfold acyclicSeg(list, NULL);
Node* y = list ;
//@ fold acyclicSeg(list, y);
while (y->next != NULL )
//@ loop_invariant acyclicSeg(list, y) &&
acc(y->next) && acc(y->val) &&
acyclicSeg(y->next, NULL) ; @*/
{
//@ unfold acyclicSeg(y, NULL);
//@ fold acyclicSeg(tmp->next, y);
//@ fold acyclicSeg(tmp, y);
mergeLemma(list, tmp, y);
//@ fold acyclic(list);
return list ;
}
//@ unfold acyclicSeg(y->next->next, NULL);
//@ fold acyclicSeg(y->next, NULL);
//@ fold acyclicSeg(y, NULL);
mergeLemma(list, y, NULL);
//@ fold acyclic(list);
return list;

Fig. 3. The incremental verification of \texttt{insertLast} from Fig. 1

To prove \texttt{acyclic(list)} holds at the end of \texttt{insertLast} (line 35), it is sufficient to prove instead that \texttt{acyclicSeg(list, NULL)} holds (line 34). After inserting a new node at the end of the list (lines 27-29), we can build up an inductive proof with folds (lines 30-32) that the list is acyclic from the insertion point \(y\) to the new end, \(i.e.\ \texttt{acyclicSeg}(y, \texttt{NULL})\) holds. We also have that the list is acyclic from the root to \(y\) (\texttt{acyclicSeg}(list, \(y\))) from to the loop invariant, and so, we are done after proving transitivity of acyclic list segments, \(e.g.\ \texttt{acyclicSeg}(list, y)\) and \texttt{acyclicSeg}(y, \texttt{NULL}) implies \texttt{acyclicSeg}(\texttt{list}, \texttt{NULL}). Sadly, static verifiers cannot automatically discharge such inductive proofs, and so we specify the proof steps in \texttt{mergeLemma} on lines 38-48. Then, after using the lemma on line 33, we achieve our proof goal.

As we can see, not only do users of static verifiers need to write a number of auxiliary specifications in support of proof goals, the specifications are often more complex compared to the program code itself even for simple examples like \texttt{insertLast}. Worse even, is that while users are developing these complex specifications static tools provide limited feedback on their correctness as demonstrated next (§2.2).

2.2 Lack of Early Specification Feedback

Since static verifiers, like Viper, limit themselves to reasoning about predicates iso-recursively and rely on loop invariants to prove properties about loops, feedback on the correctness of specifications early in the specification process is limited. For example, consider that a user named Daisy incorrectly specifies the body of \texttt{acyclicSeg} (our recursive predicate) as (\texttt{s} == \texttt{e}) ? \texttt{acc(s->val)} && \texttt{acc(s->next)} && \texttt{acyclicSeg(s->next, e)} : true, which swaps the branches of the ternary in the correct specification from Fig. 2. Let’s see how Daisy comes across this error while using Viper to incrementally specify \texttt{insertLast} in Fig. 3. Each increment from the first to the last (seventh) is highlighted in a different color. The first increment, highlighted in green, specifies the precondition and postcondition of \texttt{insertLast} with \texttt{acyclic} and \texttt{acyclicSeg} (lines 2-4). Since predicates are black boxes in static verifiers, Viper only tells Daisy that there is insufficient permission to access \(y->next\) in the loop condition on line 10. So, Daisy specifies the required \texttt{acc(y->next)} permission (as highlighted in purple for the second increment on line 12) in the loop invariant, which must frame the loop condition. But, alas Viper cannot prove that \texttt{acc(y->next)}...
holds on entry to the loop. Without realizing the branches of acyclicSeg are out of order, Daisy expects acyclicSeg(list, NULL)'s body to provide acc(y->next) at loop entry as list != NULL and y == list; and so, she unfolds acyclic(list) and acyclicSeg(list, NULL) on lines 6-7 making up the third specification increment highlighted in pink. Unfortunately, Viper still reports that acc(y->next) does not hold on entry to the loop, which alerts Daisy to the problem with acyclicSeg.

With Viper, Daisy required three specification increments to detect a bug in the first unfolding of acyclicSeg, and this problem gets worse the deeper the bug is in the recursive predicate. For example, consider now that acyclicSeg's body is incorrectly specified as (s == e) ? true : acc(s->val) & acc(s->next) & acyclicSeg(e, s->next), which swaps s->next and e in the recursive call to acyclicSeg. As a result, acyclicSeg asserts in lock-step that the nodes in lists s and e are accessible and separated—which is not the intended behavior of acyclicSeg—and acyclicSeg now always fails when e reaches its end, i.e. is NULL, as it tries to assert acc(e->val) and acc(e->next). It takes until the fourth specification increment highlighted in blue to discover that acyclicSeg is incorrectly specified using Viper. As before, Daisy is led to specifying the first three increments by Viper's error messages that first require acc(y->next) in the loop invariant and then require acc(y->next) to hold on entry to the loop. This time, however, acyclicSeg(list, NULL)'s body contains acc(y->next) when list != NULL, so Viper can prove acc(y->next) holds on loop entry and instead reports that the loop invariant acc(y->next) might not be preserved by the loop body. Daisy recalls the loop iterates over all nodes in the list with the current node being y, so preserving acc(y->next) in the loop is the same as showing that Viper has accessibility predicates for every node in the list. As a result, she specifies the fourth increment (lines 12-13 and 17), which continuously unfolds the acyclicSeg(list, NULL) predicate on every iteration of the loop and captures the information in its body in the loop invariant. Alas, Viper reports that the new loop invariant does not hold on entry as it cannot prove acyclicSeg(y->next, NULL) holds here. Since this information should come from unfolding acyclicSeg(list, NULL) on line 7, Daisy takes another look at acyclicSeg's body and discovers her specification error. That is, it takes four specification increments for Daisy to realize her mistake and the fourth increment required her to think deeply about her while loop.

Clearly, static verifiers burden their users, like Daisy, by requiring them to write a number of complex auxiliary specifications both for proofs and to receive useful feedback on the correctness of their specifications. Fortunately, as we will show next in §3, Gradual C0 overcomes this burden by smoothly supporting the spectrum between static and dynamic checking.

3 GRADUAL C0 TO THE RESCUE

In this section, we show how Gradual C0's ability to smoothly integrate static and dynamic checking allows users to overcome specification burdens inherent to static verification, e.g. that users have to write complex auxiliary specifications in support of proofs and to receive useful feedback on the correctness of their specifications.

3.1 Ignore Auxiliary Specifications with Gradual C0

In §2.1, we saw how static verifiers force users to write complex auxiliary specifications (like folds and unfolds, loop invariants, and inductive lemmas) in support of proof goals. In contrast, Gradual C0 allows users to write as many or as few auxiliary specifications as they want, and instead utilizes dynamic verification to check proof obligations not discharged statically due to missing specifications. For example, consider Fig. 3, in which our user Daisy incrementally specifies insertLast for preservation of list acyclicity and non-nullness. With Gradual C0, Daisy only needs to specify the first increment in green, which contains the pre- and postcondition of insertLast on lines 2-4. She can completely avoid specifying the auxiliary specifications in the rest
of the increments by instead specifying ? in the loop invariant on line 10. Then, Gradual C0 uses the allocation statement on line 23 to statically validate the write accesses of y->next->val and y->next->next on lines 24-25. It can also prove statically that the list after insertion is non-null. All other proof obligations, which ensure memory safety of the while loop and the list after insertion is acyclic, are checked dynamically.

Run-time checking memory safety of the while loop involves three parts: 1) verifying access to y->next in the loop condition (line 10), 2) verifying access to y->next in the loop body (line 16), and 3) verifying access to y->next in the alloc statement directly after the loop (line 23). That is, Gradual C0 asserts ownership of all heap locations in the given list, which are being iterated over by the loop. Consequently, the larger the list the higher the run-time cost of verification for insertLast. This cost is unacceptable to Daisy, so she statically specifies memory safety of the while loop in the first four increments in Fig. 3 (lines 2-4, 6-7, 12-13, and 17). The new loop invariant (lines 12-13) uses acyclicSeg to expose \texttt{acc}(y->next) for verifying access to y->next in the loop condition, loop body, and after the loop. The unfolds on lines 6-7 and 17 are used to prove that the loop invariant is preserved by the loop given the precondition on line 2. After all this work, Daisy is not interested in also statically specifying the list after insertion is acyclic, and so, she joins ? with her newly specified loop invariant. As a result, Gradual C0 now additionally checks memory safety of the while loop statically reducing run-time cost, and only checks \texttt{acyclic(result)} from the postcondition of insertLast (line 3) dynamically (i.e. the list returned after insertion is acyclic). By allowing Gradual C0 to check \texttt{acyclic(result)} dynamically, Daisy saved herself a lot of specification effort. She avoided building up acyclicSeg from the previous end of the list to the new one (increment five in orange, lines 26-28), specifying a more complex loop invariant (increment six in red, lines 9, 11, and 18-19), and stating and proving transitivity of acyclic list segments (increment seven in yellow, lines 20 and 29). Daisy is very happy to make this human-effort vs run-time cost trade-off.

### 3.2 Gradual C0 Provides Earlier Feedback with Run-Time Checks

As we saw in §2.2, static verifiers struggle to provide early feedback on specification errors in predicates. Using Viper, it took until the third specification increment in Fig. 3 for Daisy to discover the simple error in acyclicSeg’s body, (s == e) ? \texttt{acc}(s->val) &\& \texttt{acc}(s->next) &\& acyclicSeg(s->next, e): \texttt{true}, which swaps the branches of the ternary in the correct specification from Fig. 2. In contrast, with Gradual C0, Daisy easily discovers this error on the first specification increment (in green, lines 2-4), which specifies the pre- and postcondition of insertLast. She additionally specifies ? on the loop invariant and provides a simple test case that calls insertLast on a list with one node. Then, Gradual C0 alerts Daisy to the error in acyclicSeg by reporting at run time that \texttt{acc}(s->val) from acyclicSeg does not hold at the end of the list where \texttt{s == NULL}. Clearly, \texttt{acc}(s->val) will never hold when \texttt{s == NULL}, so Daisy realizes she swapped the branches in acyclicSeg. Similarly, the second example of an error in acyclicSeg’s specification, which swaps s->next and e in the recursive call to acyclicSeg ((s == e) ? \texttt{true} : \texttt{acc}(s->val) && \texttt{acc}(s->next) && acyclicSeg(e, s->next)), took four specification increments in Fig. 3 to be exposed by Viper. Again with Gradual C0, Daisy can detect the error by the first increment as long as she specifies ? on her loop invariant and supplies a simple test case with a list containing two elements. In this case, Gradual C0 reports at run time that \texttt{acc}(s->val) from acyclicSeg does not hold for \texttt{s which is NULL}. Since \texttt{acc}(s->val) will never hold when \texttt{s == NULL}, Daisy takes another look at acyclicSeg’s body and realizes her error. That is, Gradual C0’s dynamic checking of partial specifications is helpful for detecting errors in recursive predicates much earlier in the specification process than static verification alone and the errors better capture the inherent problems in the specifications.
To summarize, users of Gradual C0 may write as many or as little auxiliary specifications as they want and still get sound verification of their code by trading off between human-effort and run-time cost. Users can also receive feedback on the correctness of their specifications much earlier in the specification process than if they used static verification alone and the run-time errors reported often closely match the inherent problems with the specification.

4 GRADUAL C0’S DESIGN AND IMPLEMENTATION

Gradual C0 is a working gradual verifier for the C0 programming language [Arnold 2010] that is built as extension of the Viper static verifier [Müller et al. 2016]. Our goals for the design and implementation of Gradual C0 are to:

- be easily extensible to other programming languages beyond C0,
- minimize run-time overhead from verification as much as possible without introducing highly complex algorithms, and
- use symbolic execution for static reasoning.

Consequently, we settled on the design illustrated in Fig. 4. Gradual C0 is structured in two major sub-systems: 1) the gradual verification pipeline and 2) the C0 pipeline. Within the gradual verification pipeline, a C0 program is first translated AST-to-AST into a Gradual Viper program by Gradual C0’s frontend module, GVC0. The GVC0 module implements a simple parser, abstract syntax, and type checker for C0 programs to facilitate the translation. The Gradual Viper module is the backend of Gradual C0 and implements its own parser, abstract syntax, and type checker for its own imperative language called the Gradual Viper language. This language is the Viper language plus imprecise formulas and allows Gradual Viper to support multiple frontend languages, not just C0. We chose to build a C0 frontend first because C0 is a pedagogical version of C designed with dynamic verification in mind, and we plan to use it in the classroom.

Once the C0 program is translated into a Gradual Viper program, it is optimistically statically verified by the Gradual Viper module. Gradual Viper extends Viper’s symbolic execution based verifier to support imprecise formulas and resulting holes in static reasoning as inspired by Wise et al. [2020] and gradual typing [Herman et al. 2010; Siek and Taha 2006; Siek et al. 2015]. Consequently, by construction Gradual Viper supports full static verification of programs when specifications are complete. Differing from Wise et al. [2020]’s work, Gradual Viper also extends the symbolic execution algorithm to create a description of needed run-time checks in support of static holes. The run-time checks are minimized with statically available information during reasoning. Finally, GVC0 takes these run-time checks and encodes them in the original C0 program to produce a sound, gradually-verified program. The C0 pipeline takes this C0 program and feeds it to the C0 compiler, CC0, which is used to execute the program. Note, the encoding of checks into C0 source code optimizes for run-time performance and simplifies extending C0 with dynamic verification in our domain.

The rest of this section describes the implementation of Gradual Viper and GVC0’s designs in more detail and illustrates the concepts via example. We also highlight design and implementation choices influenced by our goals. §4.1 discusses how C0 programs are translated to Gradual Viper programs, along with modifications made to both C0 and Viper for gradual verification. Then, §4.2 and §4.3 detail Gradual Viper’s symbolic execution approach and how it produces minimized
run-time checks. Finally, §4.4 focuses on how GVC0 turns run-time checks from Gradual Viper into C0 code for dynamic verification.

4.1 Translating C0 Source Code to Gradual Viper

Source Code for Verification

The C0 language, with its minimal set of language features and its existing support for specifications, serves well as the target language for our implementation. As its name suggests, C0 borrows heavily from C, but its feature set is reduced to better suit its intended purpose as a tool in computer science education [Arnold 2010]. It is a memory-safe subset of C that forbids casts, pointer arithmetic, and pointers to stack-allocated memory. All pointers are created with heap allocation, and de-allocation is handled by a garbage collector.

The abstract syntax for C0 programs supported by Gradual C0 is given in Fig. 6, i.e. GVC0’s abstract syntax. GVC0 programs are made of struct and method declarations that largely follow C syntax. What differs from both C and C0 is GVC0’s specification language. Methods may specify constraints on their input and output values as side-effect-free gradual formulas \( \tilde{\phi} \), usually in \texttt{//}@requires or \texttt{//}@ensures clauses in the method header. Loops and abstract predicates contain invariants and bodies respectively that are made of gradual formulas. Such formulas \( \tilde{\phi} \) are imprecise
formulas \&\& \phi or complete boolean formulas \phi (Note, in this case, \phi must be self-framed as defined in IDF). A formula \phi joins boolean values, boolean operators, predicate instances, accessibility predicates, and conditionals via the separating conjunction \&\& . GVC0 programs also contain \texttt{//@fold \ p(\tilde{e})} and \texttt{//@unfold \ p(\tilde{e})} statements for predicates and \texttt{//@assert \ \phi} statements for convenience.

To support the gradual verification of many different imperative programming languages, Gradual Viper verifies programs written in its own custom imperative language, which is designed to ease the translation from other imperative languages into it. The Gradual Viper language’s abstract syntax is given in Fig. 7. The GVC0 and Gradual Viper languages are roughly 1-to-1, including their specification languages, so translation is mostly straightforward, but there are some differences as highlighted in yellow (trivial) and blue (non-trivial) in Fig. 8. For example, for loops in GVC0 are rewritten as \texttt{while} loops in Gradual Viper, and \texttt{alloc(struct \ T)} expressions are translated to new statements containing \texttt{struct \ T}’s fields. Additionally, GVC0 allows method calls, \texttt{allocs}, and ternaries in arbitrary expressions, while Gradual Viper only allows such constructs in corresponding program statements\textsuperscript{3}. Therefore, GVC0 uses fresh temporary variables to version expressions containing the aforementioned constructs into program statements in Gradual Viper. The temporary variables are then used in the original expression in place of the corresponding method call, \texttt{alloc}, or ternary. Nested field assignments, such as \(x->y->z = a\), are similarly expanded into multiple program statements using temporary variables. Value type pointers in GVC0 are rewritten as pointers to single-value structs that can be easily translated into Gradual Viper syntax. Finally, \texttt{assert(\ e)} statements are essentially ignored in Gradual Viper; \(e\) is translated into Gradual Viper syntax to verify its heap accesses, but \(e\) is not asserted. Instead, the assert is always kept in the original C0 program and is checked exclusively at run time. Fig. 9 provides a simple example program written in both the GVC0 language (Fig. 9a) and Gradual Viper language (Fig. 9b) for reference.

Note that GVC0 does not support array and string values since gradually verifying any interesting properties about such constructs requires non-trivial extensions to current gradual verification theory. Similarly, the Gradual Viper language, in contrast to the Viper language, does not support the aforementioned constructs and fractional permissions.

\textbf{4.2 Gradual Viper: Symbolic Execution for Gradual Verification}

In this section, we describe the design and implementation of Gradual Viper’s symbolic execution based algorithm supporting the static verification of imprecise formulas. Gradual verification of recursive heap data structures was formalized with weakest liberal preconditions [Wise et al. 2020]. All of the static verifiers supporting separation logic or IDF—such as Viper [Müller et al. 2016], VeriFast [Jacobs et al. 2011], JStar [Distefano and Parkinson J 2008], and SmallFoot [Berdine et al. 2006]—reason with symbolic execution. Therefore, this work serves as a first guide for building gradual verifiers from static verifiers using symbolic execution for reasoning. Additionally, unlike in Wise et al. [2020]’s work, we use our static reasoning algorithm not just for optimistic static verification but also for soundly reducing the number of run-time checks required during dynamic verification. That is, during a single execution of Gradual Viper a program is statically verified and a set of minimized run-time checks is produced for program points where the algorithm is optimistic during verification due to imprecision.

Before formalizing Gradual Viper’s implementation in §4.3, we first demonstrate at a high-level with examples how symbolic execution is used to perform optimistic static verification of programs containing recursive heap data structures and how minimized run-time checks are produced during

\textsuperscript{3}Note, ternaries correspond to if statements
this process. We also point out novel technical challenges faced and solutions developed thanks to relying on symbolic execution both for static verification and minimizing run-time checks.

4.2.1 Optimistic static verification in Gradual Viper by example. The simple program given in Fig. 9b implements a withdraw function (method), which subtracts the balance in one bank account (the subtrahend) from the balance in another account (the minuend) returning the result.\(^4\) Any client program of withdraw must ensure the subtrahend’s balance is less than or equal to the minuend’s balance and that both balances are positive as specified by withdraw’s precondition (line 11). Then, withdraw will return an account with a positive balance as specified by withdraw’s postcondition (lines 12-13). Additionally, withdraw’s postcondition ensures the subtrahend’s balance remains positive as well. Note, the withdraw example is contrived to better illustrate how Gradual Viper works and its interesting aspects.

Well-formedness of user written specifications. Gradual Viper begins static verification by first checking user written specifications, like predicate bodies, preconditions, and postconditions, for well-formedness. That is, user specifications must be self-framed and cannot contain duplicate accessibility predicates or predicates joined by the separating conjunction &&. Self-framing from IDF [Smans et al. 2009] simply means that a formula must contain accessibility predicates for any heap locations accessed in the formula. In gradual verification [Wise et al. 2020], ? can represent these accessibility predicates. For example, in the withdraw program geqTo’s body (lines 4-5) is self-framed, because ? can represent acc(a1.balance) and acc(a2.balance) to frame a1.balance and a2.balance. On the other hand, positive’s body (line 7) is classically self-framed as it explicitly contains acc(a.balance) to frame a.balance. All of the user written formulas, which are geqTo’s body (lines 4-5), positive’s body (line 7), withdraw’s precondition (line 11), and its postcondition (lines 12-13), are well-formed.

Next, Gradual Viper optimistically statically verifies each function in the given program, e.g. the withdraw function in our running example. This involves symbolically executing the function from top to bottom and tracking information in a symbolic state. Information is gathered from the execution of both specifications and code, and proof obligations are established by the symbolic state. If any obligations are established optimistically, corresponding run-time checks are stored in the symbolic state. Fig. 9c displays the contents of the symbolic state at every program point (marked by program lines) during the verification of withdraw. In general, a symbolic state can be thought of as writing an intermediate logical formula in a special form. We will discuss the contents of a symbolic state in more detail as we work through the withdraw example.

Producing a precondition. At the start of withdraw (lines 14-15), information in the precondition, e.g. geqTo(a1,a2), is produced or translated into an empty symbolic state resulting in the first state in the table in Fig. 9c. As with formulas, symbolic states may be imprecise or not, meaning information may be missing from the state due to imprecision. In fact, you can think of an imprecise symbolic state as representing an imprecise intermediate formula. Here, the precondition geqTo(a1,a2) is precise,\(^5\) so the state also remains precise. Had the precondition been imprecise, then the state would also become imprecise. Local variables are mapped to symbolic values in a symbolic, variable store. Since a1 and a2 are arguments to withdraw and res the return value, they are all assigned fresh symbolic values t1, t2, and t3 respectively in the store. Then, permissions like accessibility

\(^4\)Note, we refer to the version of the withdraw program written in the Gradual Viper language rather than its Gradual C0 counterpart in Fig. 9a as we will be discussing how Gradual Viper works in this section.

\(^5\)Note, precision in a static context as in Gradual Viper means the formula does not contain ? at the top-level. Predicates are treated as black-boxes, so even if their bodies are imprecise, as with geqTo(a1,a2), a formula containing them, such as geqTo(a1,a2), can be precise.
Gradual C0

1 struct Account { int balance; }
2 typedef struct Account Account;
3 /*@ predicate geqTo(Account* a1, Account* a2) =
4 ? & a1->balance >= a2->balance &
5 a2->balance >= 0; */
6 /*@ predicate positive(Account* a) =
7 acc(a->balance) & a->balance >= 0; */
8
9 Account* withdraw(Account* a1, Account* a2)
10 { //@ requires geqTo(a1,a2);
11   //@ ensures ? & positive(a2) &
12   positive(result);
13   
14   if (a1 == NULL || a2 == NULL) {
15     return a1;
16   } else {
17     int newB = a1->balance - a2->balance;
18     a1->balance = newB;
19     //@ fold positive(a1);
20     //@ fold positive(a2);
21     return a1;
22   }
23 }
24 }
25

(a) A simple bank withdraw example written in the Gradual C0 language

(b) A simple bank withdraw program written in the Gradual Viper language

<table>
<thead>
<tr>
<th>Ln</th>
<th>Imprecise</th>
<th>Opt. Heap</th>
<th>Heap</th>
<th>Var Store</th>
<th>Path Condition</th>
<th>Run-time Checks</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>No</td>
<td>0</td>
<td>geqTo(t1,t2)</td>
<td>a1-&gt;t1 ; a2-&gt;t2 ;</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>15-16</td>
<td>Yes</td>
<td>acc(t1,balance,p1) : acc(t2,balance,p2)</td>
<td>0</td>
<td>a1-&gt;t1 ; a2-&gt;t2 ;</td>
<td>t1 != null ; t2 != null ;</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>17</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>18-19</td>
<td>Yes</td>
<td>acc(t1,balance,p1) : acc(t2,balance,p2)</td>
<td>0</td>
<td>a1-&gt;t1 ; a2-&gt;t2 ;</td>
<td>t1 != null ; t2 != null ;</td>
<td>0</td>
</tr>
<tr>
<td>19</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>20-21</td>
<td>Yes</td>
<td>0</td>
<td>acc(t1,balance,p3)</td>
<td>a1-&gt;t1 ; a2-&gt;t2 ;</td>
<td>t1 != null ; t2 != null ;</td>
<td>0</td>
</tr>
<tr>
<td>21</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>22-23</td>
<td>Yes</td>
<td>0</td>
<td>positive(t1)</td>
<td>a1-&gt;t1 ; a2-&gt;t2 ;</td>
<td>t1 != null ; t2 != null ;</td>
<td>lbc, ~(a1 = null</td>
</tr>
<tr>
<td>23</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>24</td>
<td>lbc, ~(a1 = null</td>
<td></td>
<td>a2 = null) -&gt; l41, acc(a2.balance) ; lbc, ~(a1 = null</td>
<td></td>
<td>a2 = null) -&gt; l42, a2.balance &gt;= 0 ;</td>
<td></td>
</tr>
</tbody>
</table>

(c) Contents of the symbolic state at each program point during Gradual Viper’s static verification of withdraw in Fig. 9b

Fig. 9. A gradually verified, bank withdraw program that is contrived to illustrate how Gradual Viper works
predicates and predicates can be stored in a symbolic heap (either the optimistic heap or heap) in terms of the symbolic values. We call symbolic versions of permissions heap chunks. Since \( \text{geqTo}(a_1, a_2) \) is concretely known it is added directly to the heap as the heap chunk \( \text{geqTo}(t_1, t_2) \).

An important invariant of the heap is that permissions in it are guaranteed to be separated in memory, i.e. when they are joined by the separating conjunction they return true. The optimistic heap contains heap chunks for accessibility predicates that are optimistically assumed during verification and is introduced in this work to reduce the number of run-time checks produced by Gradual Viper. We will see how this works as we continue to discuss the withdraw example. For now, the optimistic heap is empty. Similarly, both the path condition and set of run-time checks both remain empty. The path condition contains constraints on symbolic values that have been collected on the current verification path. The precondition \( \text{geqTo}(t_1, t_2) \) only contains permission information, so the path condition is empty. Further, producing a formula into the symbolic state does not introduce any run-time checks.

### Unfolding a predicate

Next, Gradual Viper executes the unfold statement on line 15 causing the predicate \( \text{geqTo}(a_1, a_2) \) to be consumed and then its body to be produced into the state on lines 15-16. In general, consuming a formula 1) checks whether the formula is established by the symbolic state, 2) generates minimized run-time checks for the state to establish the formula soundly, and 3) removes permissions asserted in the formula from the symbolic state. That is, consume is Gradual Viper’s mechanism for checking proof obligations; and as we will see, is used a few different times throughout the verification of withdraw. Here, since \( \text{geqTo}(t_1, t_2) \) is in the heap, \( \text{geqTo}(a_1, a_2) \) is established by the symbolic state and no run-time checks are required. It is then removed from the heap as it is “consumed”. After consumption, \( \text{geqTo}(a_1, a_2) \)'s body (lines 4-5) is produced into the current state (the one without \( \text{geqTo}(t_1, t_2) \)). The body of \( \text{geqTo} \) is imprecise, so the symbolic state is made imprecise (as seen in Fig. 9c at lines 15-16). The rest of \( \text{geqTo} \)'s body is a boolean expression constraining \( a_1 \) and \( a_2 \)'s account balances: \( a_1 \)'s balance is greater than or equal to \( a_2 \)'s balance and \( a_2 \)'s balance is positive. Before adding these constraints to the path condition, Gradual Viper first looks for heap chunks in the current symbolic state corresponding to accessibility predicates that frame \( a_1.\text{balance} \) and \( a_2.\text{balance} \) in \( \text{geqTo} \)'s body. Both the heap and optimistic heap are empty, but the state is imprecise so the missing heap chunks are optimistically assumed to be in the state. In fact, it is sound to make this assumption without any run-time checks, because we are producing (rather than consuming) the predicate body. As a result, fresh symbolic values \( p_1 \) and \( p_2 \) for \( a_1.\text{balance} \) and \( a_2.\text{balance} \) respectively are generated and used to record constraints on the balances in the path condition. Gradual Viper also records that the receivers \( a_1 \) and \( a_2 \) are non-null in the path condition, because Gradual Viper assumed they can be safely de-referenced. Finally, Gradual Viper adds heap chunks \( \text{acc}(t_1, \text{balance}, p_1) \) and \( \text{acc}(t_2, \text{balance}, p_2) \) to the optimistic heap to 1) record the mappings of locations to their values and 2) avoid producing run-time checks for the accessibility predicates later in the program. These heap chunks are added to the optimistic heap rather than the heap, because \( \text{getTo} \)'s body does not specify whether or not \( a_1(t_1) \) or \( a_2(t_2) \) alias. So adding them to the heap would break the heap’s invariant. The optimistic heap, however, does not maintain any invariants. It is also convenient to store optimistic heap chunks in their own heap signaling that they are available due to optimism in the verification. The final symbolic state after consuming \( \text{geqTo}(a_1, a_2) \) and producing its body is given in Fig. 9c lines 15-16.

### Branching

After the unfold on line 15, Gradual Viper reaches the start of the if statement on the following line 16. As is common with static verifiers based on symbolic execution, Gradual Viper’s execution branches at if statements. Execution also branches at other conditioned points, such as logical conditionals and loops. Gradual Viper analyzes the then branch (lines 16-18) under the assumption the condition \( a_1 \equiv \text{null} || a_2 \equiv \text{null} \) is true, and the else branch (lines 18-24)
under the assumption \( a_1 == \) null \( \lor \) \( a_2 == \) null is false. These assumptions are added to the path condition for each execution path respectively. However, in our example the symbolic state going into the if statement (Fig. 9c lines 15-16) states that both \( a_1 \) and \( a_2 \) are non-null. So the then branch is infeasible, and Gradual Viper prunes this execution path resulting in the blank symbolic states in Fig. 9c from lines 16-18. Both accounts being non-null means the else branch condition for sure holds and so execution proceeds down this branch without any changes to the symbolic state. That is, for withdraw to be statically verified, this one execution path must successfully verify. If Gradual Viper executed both branches, then determining verification success is a bit more complicated:

- If the current symbolic state is precise, then both execution paths must successfully verify. This is the default functionality in static verifiers.
- If the current symbolic state is imprecise, then verification succeeds when one or both paths successfully verify. When only one path succeeds and the state is imprecise, Gradual Viper optimistically assumes the state contains information that forces program execution down the success path only at run time. This more permissive static functionality is critical for adhering to the gradual guarantee at branch points. To ensure the program will never actually execute the failing branch (i.e., to ensure soundness), Gradual Viper adds a run-time check for the success path’s condition at the branch point.

**Variable assignment.** Let’s look now at how Gradual Viper verifies the else branch (lines 18-24). At the variable assignment on line 19, Gradual Viper first evaluates the right-hand expression to the symbolic value \( p_1 - p_2 \). To do this, Gradual Viper first looks for heap chunks for \( a_1.balance \) and \( a_2.balance \) in the current symbolic heaps (Fig. 9c lines 18-19) to both frame the locations and get their values. Both heap chunks are in the optimistic heap, so no run-time checks are required for framing and \( p_1 \) and \( p_2 \) are used in the evaluation of the right-hand expression. Note, if Gradual Viper did not add the aforementioned heap chunks to the optimistic heap when producing the body of \( \text{geqTo} (a_1,a_2) \), then Gradual Viper would create run-time checks for them here in the program. However, these checks would be duplicates, because Gradual Viper also checks that these heap chunks are available when ensuring the precondition \( \text{getTo} (a_1,a_2) \) holds in client contexts at calls to withdraw. So sound tracking of heap chunks in an optimistic heap has helped us avoid duplicating run-time checks! Finally, Gradual Viper adds a new mapping to the variable store for \( \text{newB} \) and its fresh symbolic value \( t_4 \); and then, adds the constraint \( t_4 = p_1 - p_2 \) to the path condition to record information from the assignment in the symbolic state (Fig. 9c lines 19-20).

**Field assignment.** Next, \( a_1.balance \) is assigned \( \text{newB} \)’s value in the field assignment on line 20. Gradual Viper mimics this behavior symbolically by first pulling \( \text{newB} \)’s value \( t_4 \) from the symbolic state (Fig. 9c lines 19-20). Then, Gradual Viper looks for \( a_1.balance \)’s heap chunk in the state for framing, and if there, removes the chunk as \( a_1.balance \)’s value may change in the write, i.e. \( \text{acc}(a_1.balance) \) is consumed. Gradual Viper also asserts that \( a_1 \) is non-null. The heap chunk for \( a_1.balance \) is in the optimistic heap and \( a_1 != \) NULL is in the path condition, so no run-time checks are needed here. Then, \( a_1.balance \)’s heap chunk is removed from the state; and, unfortunately, this action causes \( a_2.balance \)’s heap chunk to be removed from the state as well. Gradual Viper does not know whether or not \( a_1 \) and \( a_2 \) alias, because this information does not appear in the current path condition and the optimistic heap does not maintain the separation invariant. Then, since the state is imprecise Gradual Viper could assume that \( a_1.balance \) and \( a_2.balance \) refer to the same heap location, i.e. \( a_1 \) and \( a_2 \) are aliased. In this case, removing \( a_1.balance \)’s heap chunk requires also removing \( a_2.balance \)’s heap chunk as \( a_2.balance \) may have also changed with the write. On the other hand, the case where they do not alias and \( a_2.balance \)’s heap chunk can stay in the optimistic heap is also possible from Gradual Viper’s perspective. To simply and soundly cover
both cases Gradual Viper removes \(a2.balance\)'s heap chunk by default when alias information is unknown. As we will see next, this comes at the cost of additional run-time checks later in the verification of withdraw. Finally, Gradual Viper produces a new heap chunk for \(a1.balance\) into the state to track its new, fresh symbolic value \(p3\) after the write and updates the path condition with the assignment information \(p3 = t4\) (Fig. 9c lines 20-21).

**Folding a predicate.** After \(a1.balance\) is assigned a new balance, Gradual Viper executes the fold statement on line 21. Folding a predicate is similar to unfolding a predicate except that the functionality is reversed: \(\text{positive(a1)}\)'s body is consumed from the current state (Fig. 9c lines 20-21) and then \(\text{positive(a1)}\) is produced into the state after consumption (Fig. 9c lines 21-22). The body of \(\text{positive(a1)}\) is \(\text{acc(a1.balance)} \&\& a1.balance >= 0\), so \(\text{acc(a1.balance)}\) is consumed first then \(a1.balance \geq 0\) second. The heap chunk \(\text{acc(t1,balance,p3)}\) corresponding to \(\text{acc(a1.balance)}\) is in the heap and \(a1 != \text{null}\) holds in the path condition, so no run-time check is required for consuming \(\text{acc(a1.balance)}\). Then, \(a1.balance\)'s heap chunk is removed from the heap as seen in Fig. 9c lines 21-22. Next, the boolean expression \(a1.balance >= 0\) is evaluated to its symbolic value \(p3 >= 0\). Recall that to do this, Gradual Viper must look up a heap chunk for \(a1.balance\) in the symbolic state to both frame the heap location and get its value. However, Gradual Viper just removed this heap chunk from the current state due to the left-to-right execution of consume. To solve this issue, Gradual Viper looks for framing and value information in the state before the fold, i.e. the state before consumption at lines 20-21 in Fig. 9c. As we know, \(a1.balance\)'s heap chunk is in this state and maps \(a1.balance\) to the value \(p3\), so no run-time check is needed for framing. Then, \(p3 >= 0\) is asserted against the current path condition. Since \(p1 >= p2 >= 0\) and \(p3 = t4 = p1 - p2\) are in the path condition, \(p3\) is clearly greater than or equal to 0 and is proven directly by the path condition. That is, no run-time check is needed for \(p3 >= 0\). Finally, \(\text{positive(a1)}\) is produced into the current state, which adds \(\text{positive(t1)}\) to the heap resulting in the final version of the state in Fig. 9c lines 21-22.

Next, Gradual Viper executes the second fold statement on line 22 with the aforementioned state. This fold statement consumes \(\text{positive(a2)}\)'s body and then produces \(\text{positive(a2)}\) into the state. So as before, Gradual Viper first consumes \(\text{acc(a2.balance)}\) and then \(a2.balance \geq 0\). The receiver \(a2\) is proved to be non-null by the path condition; however this time, the heap chunk for \(\text{acc(a2.balance)}\) is not in either of the heaps. Fortunately for us, the state is imprecise and can optimistically contain this heap chunk, so a run-time check is produced for \(\text{acc(a2.balance)}\) as seen in the state after folding \(\text{positive(a2)}\) in Fig. 9c lines 22-23. Since this run-time check occurs down the else branch of the if statement in withdraw, branch information, e.g. \(l_{bc}, \neg(a1 = \text{null} || a2 = \text{null})\), is included with the check. The location \(l_{bc}\) specifies where the branch point originated in the program, e.g. line 16, and \(\neg(a1 = \text{null} || a2 = \text{null})\) is the assumption made at the branch point for the current execution path. Additionally, \(l_{c1}\) contains the location where the check itself is required in the program, e.g. line 22. While it does not happen in the withdraw example, sometimes different checks are required at the same program point down different execution paths. So, Gradual Viper attaches branch information for the entire execution path to each run-time check to allow GVC0 (or other frontends) to apply checks only on the execution path they are required. This ensures soundness and reduces run-time checking. Now, Gradual Viper removes \(\text{acc(a2.balance)}\) from the current state (Fig. 9c lines 21-22), which actually causes \(\text{positive(t1)}\) to be removed from the heap as well. Predicates are treated as black boxes in Gradual Viper; so unless told otherwise, Gradual Viper conservatively assumes \(\text{acc(a2.balance)}\) is in \(\text{positive(t1)}\) and removes \(\text{positive(t1)}\) from the heap alongside \(\text{acc(a2.balance)}\). The only way Gradual Viper can guarantee \(\text{acc(a2.balance)}\) is not in \(\text{positive(t1)}\) is if a heap chunk for \(a2.balance\) and \(\text{positive(t1)}\) both exist in the heap, as the heap maintains the separation invariant. In this case, \(\text{positive(t1)}\) can remain in the heap.
while only acc(a2.balance) is removed. Of course, in our example the heap chunk for a2.balance is definitely not in the heap, so positive(t1) is removed.

Continuing, Gradual Viper consumes a2.balance >= 0, which first looks for a heap chunk to frame a2.balance in the state before the consume (Fig. 9c lines 21-22). However, neither of the heaps contain a heap chunk for a2.balance. As before, Gradual Viper uses imprecision to optimistically assume the heap chunk is in the state and produces a run-time check for acc(a2.balance). Since this run-time check for the same location already exists in the state, the two checks are condensed into the first one. Then, Gradual Viper returns a fresh symbolic value for a2.balance, say p4, to evaluate a2.balance >= 0 down to p4 >= 0. Note, Gradual Viper can only return a fresh value here, because the heaps do not contain a heap chunk recording a2.balance’s value in the state. Unfortunately, this means Gradual Viper cannot prove a2.balance >= 0 holds as no constraints exist for p4 in the path condition. But, this also means p4 >= 0 does not contradict existing information in the path condition. So, imprecision in the state can optimistically represent p4 >= 0 and a run-time check for a2.balance >= 0 is generated as seen in Fig. 9c lines 22-23. A few things of note here:

- Run-time checks are originally computed in terms of symbolic values, e.g. p4 >= 0, but are ultimately replaced with counterparts written in terms of program variables, e.g. a2.balance >= 0. This replacement by the translate function in Gradual Viper simplifies the implementation of run-time checks for frontends like GVC0, which operate on program variables and concrete values not symbolic values. The translate function uses mappings in the symbolic heaps and store to reverse the symbolic execution. Special considerations are made for fresh symbolic values like p4, aliasing between object values, and different variable contexts.

- On another note, if consuming acc(a1.balance) at the field assignment on line 20 did not also consume the heap chunk for a2.balance, then the run-time checks for acc(a2.balance) and a2.balance >= 0 would not be necessary. Gradual Viper conservatively assumed a1 and a2 were aliased at the consume, so it removed both chunks from the state. However, in practice a1 and a2 are likely to be distinct objects; and in fact, folding positive(a1) then positive(a2) is a good sign the developer of withdraw expects a1 and a2 to be distinct. In this case, a2.balance’s heap chunk does not need to be removed making the aforementioned run-time checks unnecessary. Unfortunately, since we designed Gradual Viper to be conservative for simplicity, these run-time checks are only eliminated when the developer explicitly specifies that a1 and a2 are not aliased, such as in the precondition of withdraw. So, we are trading more optimal run-time checks for simplicity in our consume algorithm.

- While it does not happen here in our withdraw example, there may be times where parts of a symbolic, boolean expression are proven statically and the rest optimistically. In this case, Gradual Viper re-writes the expression into conjunctive normal form and computes the conjuncts in this form that cannot be proven statically by the path condition. These conjuncts (after translation) will then be checked at run time. We call this process computing the difference between the expression and the path condition, and it results in minimized run-time checks given statically available information.

Finally, a heap chunk for positive(a2) (e.g. positive(t2)) is produced into the heap resulting in the final form of the state in Fig. 9c lines 22-23.

Return value assignment. Then, Gradual Viper reaches the variable assignment on line 23, which assigns a1 to res—the return value of withdraw. Gradual Viper first looks up the symbolic value t1 for a1 and then the symbolic value t3 for res in the variable store. Gradual Viper stores the information t3 = t1 from the assignment in the path condition resulting in the next symbolic state in Fig. 9c lines 23-24.
Consuming a postcondition. Finally, Gradual Viper reaches the end of withdraw down its one and only execution path on line 24. So the last thing Gradual Viper must do to verify the function, is to consume the postcondition \( \text{positive} \) (lines 12-13) in the current symbolic state (Fig. 9c lines 23-24). Gradual Viper begins by first consuming positive(a2) then positive(res). The heap chunk for positive(a2) is in the heap, so no run-time check is needed for it. Then positive(t2) is removed from the heap leaving both symbolic heaps empty. As a result (and because the state is imprecise), consuming positive(res) in the next step results in a run-time check for the predicate as seen in the final set of run-time checks required for withdraw given in Fig. 9c line 24. Note, consuming \( \text{positive}(a2) \) \&\& \( \text{positive}(\text{res}) \) requires both consuming the predicates individually (which we’ve done) and ensuring that one predicate does not access heap locations overlapping with the other (in adherence with the separating conjunction \&\&). Unfortunately, the state does not contain enough information to prove this fact statically, e.g. only the heap chunk for positive(a2) appears in the heap, but the state is imprecise! So when Gradual Viper optimistically assumes positive(res) holds, it also assumes positive(res) is separated from positive(a2). Gradual Viper flags positive(res)’s run-time check with this additional check for GVC0 to handle. Additionally, after consuming the static part of an imprecise formula, e.g. \( \text{acc}(\text{positive}(a2)) \) \&\& \( \text{acc}(\text{positive}(\text{res})) \) in withdraw’s postcondition, Gradual Viper makes the state imprecise and empties both symbolic heaps. The ? in the imprecise formula can represent any permission available in the state, so they must be removed by consume.

Takeaways. To summarize, Gradual Viper statically verifies the withdraw function successfully, and produces run-time checks for \( \text{acc}(a2.\text{balance}) \) before line 20, \( a2.\text{balance} \geq 0 \) also before line 20, and positive(res) at the end of withdraw (line 24). The withdraw function will be completely verified if these checks succeed at run time. During our discussion of the withdraw function, we highlighted a number of technical challenges addressed and solutions developed related to designing and implementing Gradual Viper. One of our goals was for Gradual Viper to minimize run-time checks as much as possible without using highly complex algorithms. For this we introduced the optimistic heap, which tracks heap chunks that are optimistically assumed during static verification and can be soundly used to reduce run-time checking in successive program statements from where they originated. In withdraw, we saw the heap chunks for a1.balance and a2.balance, which were added to the optimistic heap during the production of \( \text{geqTo}(a1,a2) \)’s body (line 15), be used to eliminate duplicate run-time checks at the assignment on line 19. We had to make careful considerations for the separating conjunction and removal of heap chunks at consumes to ensure sound tracking of heap chunks in the optimistic heap. We also defined and implemented the diff function, which utilizes conjunctive normal form to optimize run-time checks for boolean expressions. Finally, Gradual Viper conservatively removes heap chunks from the symbolic heaps that may alias with other heap chunks removed at a consume. We saw in withdraw that this comes at the cost of additional run-time checks: consuming a1.balance’s heap chunk at the field assignment on line 20 also consumed a2.balance’s heap chunk resulting in run-time checks for \( \text{acc}(a2.\text{balance}) \) and a2.balance \( \geq 0 \) before line 20. That is, we are trading more optimal run-time checks for simplicity in our consume algorithm.

Another goal for Gradual Viper, is for it to use symbolic execution for static reasoning. We accomplished this goal, but not without dealing with some technical challenges. Symbolic execution based static verifiers generate and discharge proof obligations written in terms of symbolic values, causing Gradual Viper, which extends this system, to follow suit. As a result, Gradual Viper naturally generates run-time checks written in terms of symbolic values as well. Unfortunately, dynamic verifiers only operate on program variables and concrete values not symbolic ones. To bridge this gap between the static and dynamic systems, we implemented a translate function in Gradual Viper.
that re-writes run-time checks containing symbolic values to ones containing program variables and concrete values while being careful about aliases. Finally, execution splitting at branch points led to some trickiness in gradual verification. Different run-time checks may appear at the same program point along different execution paths, so we augmented Gradual Viper to attach branching information to run-time checks. We also augmented Gradual Viper to be more optimistic about verification success when dealing with failing execution paths in the presence of imprecision. This was done in compliance with the gradual guarantee [Wise et al. 2020].

4.3 Gradual Viper: Implemented Algorithm

In this section, we formalize the symbolic execution algorithm implemented by Gradual Viper. A high-level description of how it works is given in §4.2. Our algorithm extends Viper’s symbolic execution algorithm, and so Gradual Viper’s design is heavily influenced by Müller et al. [2016]’s work. Like Viper, Gradual Viper’s algorithm consists of 4 major functions: eval, produce, consume, and exec. The functions evaluate expressions, produce (inhale) and consume (exhale) formulas, and execute program statements respectively. Following Viper’s lead, our 4 functions are defined in continuation-passing style, where the last argument of each of the aforementioned functions is a continuation Q. The continuation is a function that represents the remaining symbolic execution that still needs to be performed. Note that the last continuation returns a boolean (\(\lambda \_\_\_. \text{success()}\) or \(\lambda \_\_\_. \text{failure()}\)), indicating whether or not symbolic execution was successful.

The rest of this section is outlined as follows. Run-time checks and the collections that hold them are described in §4.3.1. We define symbolic states in §4.3.2 and preliminaries in §4.3.3. Finally, the 4 major functions of our algorithm are given in their own sections: eval §4.3.4, produce §4.3.5, consume §4.3.6, and exec §4.3.7. Throughout this section, we make clear where Viper has been extended to support imprecise formulas with yellow highlighting in figures. We also use blue highlighting to indicate extensions for run-time check generation and collection.

4.3.1 Run-time checks

Run-time checks produced by Gradual Viper are collected in the \(\mathfrak{R}\) set. A run-time check is a 4-tuple \((\text{bcs}_e, \text{origin}_e, \text{location}_e, \phi_e)\), where \text{bcs}_e is a set of branch conditions, \text{origin}_e and \text{location}_e denote where the run-time check is required in the program, and \(\phi_e\) is what must be checked. A branch condition in \text{bcs}_e is also a tuple of \((\text{origin}_e, \text{location}_e, e)\), where \text{origin}_e and \text{location}_e define the program location at which Gradual Viper’s execution branches on the condition \(e\). A location is the AST element in the program where the branch or check occurs, denoted as a formula \(\phi_l\). Sometimes, the condition being checked is defined elsewhere in the program (e.g. in the precondition of a method) but we need to relate it to the method being verified. The \text{origin} is used to do this. It is none when the condition is in the method being verified; otherwise, it contains a method call, fold, unfold, or special loop statement from the method being verified that referenced the check specified in the location. An example run-time check is: \(((\text{none}, x > 2, \neg(x > 2))), z := m(y), \text{acc}(y.f), \text{acc}(y.f))\). The check is for accessing \(y.f\), and it is required for \(m\)’s precondition element \text{acc}(y.f) at the method call statement \(z := m(y)\). The check is only required when \(\neg(x > 2)\), which is evaluated at the program point where the AST element \(x > 2\) exists. Since \(\neg(x > 2)\)’s \text{origin} is none, it comes from an if or assert statement.

Further, \(\mathcal{R}\) is used to collect run-time checks down a particular execution path in Gradual Viper. \(\mathcal{R}\) is a 3-tuple \((\text{bcs}_p, \text{origin}_p, \text{rcs}_p)\) where \text{bcs}_p is the set of branch conditions collected down the execution path \(p\), \text{origin}_p is the current origin that is set and reset during execution, and \text{rcs} is the set of run-time checks collected down \(p\). Two auxiliary functions are used to modify \(\mathcal{R}\): \text{addcheck} and \text{addbc}. The \text{addcheck} function takes an \(\mathcal{R}\) collection \(\mathcal{R}_{\text{arg}}\), a \text{location} \(\phi_l\) for a check, and the check itself, and returns a copy of \(\mathcal{R}_{\text{arg}}\) with the run-time check added to \(\mathcal{R}_{\text{arg}}.\text{rcs}\). If necessary, \text{addcheck} uses \(\mathcal{R}_{\text{arg}}.\text{origin}\) and substitution to ensure \(\phi_l\) and the check refer to the

\(\mathcal{R}\) being checked.
correct context. For example, let $\phi_t$ and check $\phi_c$ come from asserting a precondition for $z := m(y)$. Then, addcheck performs the substitutions: $\phi_t[t \mapsto m_{\text{arg}}]$ (precondition declaration context) and $\phi_c[t \mapsto y]$ (method call context) where $t$ is the symbolic value for $y$. The addbc function operates similarly to addcheck but for branch conditions.

### 4.3.2 Symbolic State

We use $\sigma \in \Sigma$ to denote a symbolic state, which is a 6-tuple $(\text{isImprecise}, h_?, h, y, \pi, R)$ consisting of a boolean $\text{isImprecise}$, a symbolic heap $h_?$, another symbolic heap $h$, a symbolic store $y$, a path condition $\pi$, and a collection $R$ (defined in §4.3.1). The boolean $\text{isImprecise}$ records whether or not the state is imprecise, the symbolic store $y$ maps local variables to their symbolic values, and the path condition $\pi$ (defined in §4.3.3) contains constraints on symbolic values that have been collected on the current verification path.

A symbolic heap is a multiset of heap chunks for fields or predicates that are currently accessible. A field chunk $id(r; \delta)$ (representing expression $r.id$) consists of the field name $id$, the receiver's symbolic value $r$, and the field's symbolic value $\delta$—also referred to as the snapshot of a heap chunk. For a predicate chunk $id(args; \delta)$, $id$ is the predicate name, $args$ is a list of symbolic values that are arguments to the predicate, and $\delta$ is the snapshot of the predicate. A predicate's snapshot represents the values of the heap locations abstracted over by the predicate. The symbolic, optimistic heap $h_?$ contains heap chunks that are accessible due to optimism in the symbolic execution, while $h$ contains heap chunks that are statically accessible. Further, only $h$ maintains the invariant that its heap chunks are separated in memory, and thus, can be joined successfully by the separating conjunction. The empty symbolic state is

$$\sigma_0 = (\text{isImprecise} := \text{false}, h_? := \emptyset, h := \emptyset, y := \emptyset, \pi := \emptyset, R := (\emptyset, \text{none}, \emptyset)).$$

### 4.3.3 Preliminaries

We introduce a few preliminary definitions here that will be helpful later. A path condition $\pi$ is a stack of tuples $(id, bc, pcs)$. An $id$ is a unique identifier that determines the constraints on symbolic values that have been collected between two branch points in execution. The $bc$ entry is the symbolic value for the branch condition from the first of two branch points, and $pcs$ is the set of constraints that have been collected. Branch points can be from if statements and logical conditionals in formulas. Functions pc-all, pc-add, and pc-push manipulate path conditions and are formally defined in Appendix Fig. 21. The pc-all function collects and returns all the constraints in $\pi$, pc-add adds a new constraint to $\pi$, and pc-push adds a new stack entry to $\pi$. Similarly, snapshots for heap chunks have their own related functions: unit, pair, first, and second. The constant unit is the empty snapshot, pair constructs pairs of snapshots, and first and second deconstruct pairs of snapshots into their sub-parts. Further, fresh is used to create fresh snapshots, symbolic values, and other identifiers depending on the context. The havoc function similarly updates a symbolic store by assigning a fresh symbolic value to each variable in a given collection of variables. Finally, check$(\pi, t) = \text{pc-all}(\pi) \Rightarrow t$ queries the underlying SAT solver to see if the given constraint $t$ is valid in a given path condition $\pi$ (i.e. $\pi$ proves or implies $t$).

### 4.3.4 Symbolic execution of expressions

The symbolic execution of expressions by the eval function is defined in Fig. 10. Using the current symbolic state, eval evaluates an expression to a symbolic value $t$ and returns $t$ and the current state to the continuation $Q$. Variable values are looked up in the symbolic store and returned. For $op(\overline{e})$, its arguments $\overline{e}$ are each evaluated to their symbolic values $\overline{t}$. A symbolic value $op'(\overline{t})$ is then created and returned with the state after evaluation. Each $op$ has a corresponding symbolic value $op'$ of the same arity. For example, $e_1 + e_2$ results in the symbolic value $add(t_1, t_2)$ where $e_1$ and $e_2$ evaluate to $t_1$ and $t_2$ respectively.

Finally, the most interesting rule is for fields $e.f$. The receiver $e$ is first evaluated to $t$ resulting in a new state $\sigma_2$. Then, eval looks for a heap chunk for $t.f$ first in the current heap $h$. If a chunk exists,

---

6Heap lookup in eval also looks for heap chunks that are aliases (according to the path condition) to the chunk in question.
eval(σ, t, Q) = Q(σ, t)

if (∃ f(r; δ) ∈ σ₂, h . check(σ₂, π, r = t)) then
  Q(σ₂, δ)

else if (∃ f(r; δ) ∈ σ₂, h₂ . check(σ₂, π, r = t)) then
  Q(σ₂, δ)

else if (σ₂.isImprecise) then
  res₂ := assert(σ₂.isImprecise, σ₂, π, t ≠ null)
  e₂ := translate(σ₂, t)
  R' := addcheck(σ₂, R, e₂, acc(e₂, f))
  δ := fresh
  res ∧ Q(σ₂, h² := σ₂, h₁ ∪ f(t; δ), π := pc-add(σ₂, π, t ≠ null)), R := R', δ)
else failure())

Fig. 10. Rules for symbolically executing expressions

then the heap read succeeds and σ₂ and the chunk’s snapshot δ is returned to the continuation. If a chunk does not exist in h, then eval looks for a chunk in the optimistic heap h₂, and if found the chunk’s snapshot is returned with σ₂. If a heap chunk for t.f is not found in either heap, then the heap read can still succeed when σ₂ is imprecise. As long as t ≠ null does not contradict the current path condition σ₂,π (the call to assert, Appendix Fig. 28), σ₂’s imprecision optimistically provides access to t.f. Therefore, a run-time check for acc(e₂, f) is created and added to σ₂’s set of run-time checks (highlighted in blue). Note that e₂, f is used in the check rather than t, f, because—unlike t which is a symbolic value—the expression e₂ can be evaluated at run time. Specifically, translate (described in Appendix Fig. 25) is called on t with the current state σ₂ to compute e₂. Additionally, the AST element e.f is used to denote the check’s location.

Afterwards, a fresh snapshot δ is created for t.f’s value, and a heap chunk f(t; δ) for t.f and δ is created and added to σ₂’s optimistic heap passed to the continuation. Similarly, the constraint t ≠ null is added to σ₂’s path condition. By adding f(t; δ) to the optimistic heap, the following accesses of t.f are statically verified by the optimistic heap, which reduces the number of run-time checks produced. Finally, verification of the heap read for t.f fails when none of the aforementioned cases are true. Fig. 22 and Fig. 23 in the Appendix define variants of eval, called eval-p and eval-c, that are used in produce and consume respectively. The eval-p variant does not introduce run-time checks and eval-c does not extend the optimistic heap and path condition, because the aforementioned functionalities are not needed in these contexts.

### 4.3.5 Symbolic production of formulas

Produce (Fig. 11) is responsible for adding information to the symbolic state, in particular, the path condition and the heap h. Producing an imprecise formula makes the symbolic state imprecise. The produce rule for an expression e evaluates e to its symbolic value and produces it into the path condition. The produce rules for accessibility predicates containing fields and predicates are similar, so we focus on the rule for fields only.

The field e.f in acc(e,f) first has its receiver e evaluated to a symbolic value t. Then, using the parameter δ a fresh heap chunk f(t; δ) is created and added to the heap before invoking the
produce(σ, ? & ϕ, δ, Q) = produce(σ(isImprecise := true), ϕ, second(δ), Q)
produce(σ₁, e, δ, Q) = eval-p(σ₁, e, (λ s₂, t . Q(s₂{π := pc-add(s₂, π, \{t, δ = unit\}))))
produce(σ₁, acc(p(Π)), δ, Q) = eval-p(σ₁, Π, (λ s₂, T . Q(s₂{h := s₂.h ∪ p(Π, δ)})))
produce(σ₁, acc(e.f), δ, Q) = eval-p(σ₁, e, (λ s₂, t . Q(s₂{h := s₂.h ∪ f(t, δ), π := pc-add(s₂, π, \{t ≠ null\}))))
produce(σ₁, ϕ₁ & ϕ₂, δ, Q) = produce(σ₁, ϕ₁, first(δ), (λ s₂ . produce(σ₂, ϕ₂, second(δ), Q)))
produce(σ₁, e ? ϕ₁ : ϕ₂, δ, Q) = eval-p(σ₁, e, (λ s₂, t . branch(s₂, e, t, (λ σ₃ . produce(σ₃, ϕ₁, δ, Q)), (λ σ₃ . produce(σ₃, ϕ₂, δ, Q))))

Fig. 11. Rules for symbolically producing formulas

continuation. Note, the disjoint union ∪ ensures \( f(t; δ) \) is not already in the heap before \( f(t; δ) \) is added; otherwise, verification fails. Further, \( \text{acc}(e.f) \) implies \( e ≠ \text{null} \) and so that fact is recorded in the path condition as \( t ≠ \text{null} \). When the separating conjunction \( ϕ₁ & ϕ₂ \) is produced, \( ϕ₁ \) is first produced into the symbolic state, followed by \( ϕ₂ \). Finally, to produce a conditional, Gradual Viper branches on the symbolic value \( t \) for the condition \( e \) splitting execution along two different paths. Along one path \( ϕ₁ \) is produced into the state under the assumption that \( t \) is true, and along the other path \( ϕ₂ \) is produced under the \( ¬t \) assumption. Both paths follow the continuation to the end of its execution, and a branch condition corresponding to the \( t \) assumption made is added to the symbolic state. Paths are pruned when they are infeasible (the assumption about \( t \) would contradict the current path conditions). Overall verification success is computed from the results of the two execution paths, and an imprecise state allows this computation to be optimistic when one path successfully verifies and the other doesn’t. In this case, branch optimistically marks verification a success when normally it should fail, because the state may optimistically contain information that prunes the failure case. A run-time check is then added for the success path’s condition to ensure soundness. This functionality is important for adhering to the gradual guarantee [Wise et al. 2020]. The formal definition of branch is in Fig. 12, and other details for branch and produce are given in Appendix §A.2. Note, produce only adds run-time checks for branching to the symbolic state.

4.3.6 **Symbolic consumption of formulas.** The goals of consume are 3-fold: 1) given a symbolic state \( σ \) and formula \( \overline{ϕ} \), check whether \( \overline{ϕ} \) is established by \( σ \), i.e. \( \overline{ϕ}_σ \models \overline{ϕ} \) where \( \overline{ϕ}_σ \) is the formula which represents the state \( σ \), 2) produce and collect run-time checks that are minimally sufficient for \( σ \) to establish \( \overline{ϕ} \) soundly, and 3) remove accessibility predicates and predicates that are asserted in \( \overline{ϕ} \) from \( σ \). The rules for consume are given in full and described in great detail in Appendix §A.3. We give select rules in Fig. 13 and an abstract description here.

The functionality of consume is split across two functions: consume, which is the interface to consume accepting only a state, formula, and continuation, and consume’, which is a helper function performing consume’s major functionality. Note, before calling consume’, consume first adds non-alias information from the heap to the path condition and checks that the heap and path condition are non-contradictory using consolidate [Schwerhoff 2016].

Then, the two functions work together to accomplish the aforementioned goals. For the first and second goals, heap chunks representing accessibility predicates and predicates in \( \overline{ϕ} \) are looked up in the heap \( h \) and optimistic heap \( h_\text{opt} \) from \( σ \). When \( σ \) is precise, the heap chunks must be in \( h \) or verification fails. If \( σ \) is imprecise, then the heap chunks are always justified either by the heaps or imprecision. Run-time checks for heap chunks that are verified by imprecision are collected in \( σ.R \). The consume’ rule for \( \text{acc}(e.f) \) (and the rule for \( \text{acc}(p(\overline{ϕ})) \) which is similar) supports this

### Fig. 11. Rules for symbolically producing formulas

- **Handles imprecision**
- **Handles run-time check generation and collection**
branch(σ, e, t, Q_t, Q_{¬t}) =

\begin{align*}
(π_T, R_T) & := (\text{pc-push}(σ, π, \text{fresh}, t), \text{addbc}(σ, R, e, e)) \\
(π_F, R_F) & := (\text{pc-push}(σ, π, \text{fresh}, ¬t), \text{addbc}(σ, R, e, ¬e)) \\
\text{if (σ.isImprecise)} \text{ then} & \\
\text{rest}_T & := (\text{if } ¬\text{check}(σ, π, ¬t) \text{ then } Q_t(σ{π := π_T}, R := R_T)) \text{ else failure()} \\
\text{rest}_F & := (\text{if } ¬\text{check}(σ, π, t) \text{ then } Q_{¬t}(σ{π := π_F}, R := R_F)) \text{ else failure()} \\
\text{if } ((\text{rest}_T \land ¬\text{rest}_F) \lor (¬\text{rest}_T \land \text{rest}_F)) \text{ then} & \\
R' & := \text{addcheck}(σ, R, e, (\text{if rest} \text{ then } e \text{ else } ¬e)) \\
R & := R \cup R'.\text{last} \\
\text{rest} \lor \text{rest}_T & \\
\text{else} & \\
(\text{if } ¬\text{check}(σ, π, ¬t) \text{ then } Q_t(σ{π := π_T}, R := R_T)) \text{ else success()} & \land \\
(\text{if } ¬\text{check}(σ, π, t) \text{ then } Q_{¬t}(σ{π := π_F}, R := R_F)) \text{ else success()} &
\end{align*}

Fig. 12. Formally defining the branch function

functionality by calling heap-rem-acc (defined in Appendix Fig. 27) for the look-up, assigning the boolean results to \( b_1 \) and \( b_2 \), and then using them in if-then-else and else-if-else casing. The blue highlighting in the isImprecise is true case in the aforementioned rule handles the run-time checks. Clauses in \( \phi \) containing logical expressions are first evaluated to a symbolic value \( t \), which is then checked against \( σ' \)’s path condition \( π \). If \( σ \) is precise, then \( \text{pc-all}(π) \Rightarrow t \) must hold (i.e. the constraints in \( π \) prove \( t \)) or verification fails. In contrast, when \( σ \) is imprecise, \( \land \text{pc-all}(π) \land t \) must hold (i.e. \( t \) does not contradict constraints in \( π \)) otherwise verification fails. In this case, a run-time check is added to \( σ.R \) for the set of residual symbolic values in \( t \) that cannot be proved statically by \( π \). The consume’ rules for expressions and symbolic values implement this behavior. The call to assert (defined in Appendix Fig. 28) checks \( t \) against \( π \) and returns the result and any residual symbolic values. Note, assert uses diff from Appendix §A.1 to compute the residuals. The part highlighted in blue adds the run-time check for the residuals to the state. Finally, fields used in \( \phi \) must have corresponding heap chunks in \( h \) when \( σ \) and \( \phi \) are precise; otherwise when \( σ \) or \( \phi \) are imprecise, field access can be justified by either the heaps or imprecision. A run-time check containing an accessibility predicate for the field is added to \( σ.R \) when imprecision is relied on. This is all handled by the second argument \( f_1 \) to consume’ and eval-c called by consume’ on expressions.

The third goal of consume is to remove heap chunks \( h_{ci} \) representing accessibility predicates and predicates in \( \phi \) from \( σ \), and in particular, from heaps \( h \) and \( h_γ \). When \( σ \) and \( \phi \) are both precise, the heap chunks in \( h_{ci} \) are each removed from \( h \) (\( h_γ \) is empty here). If \( \phi \) is imprecise, then all heap chunks in both heaps are removed as they may be in \( h_{ci} \) or \( \phi \) may represent them with imprecision. Finally, when \( σ \) is imprecise and \( \phi \) is precise, any heap chunks in \( h \) or \( h_γ \) that overlap with or may potentially overlap with (thanks to \( σ’ \)’s imprecision) heap chunks in \( h_{ci} \) are removed. The calls to heap-rem-acc (and its counterpart heap-rem-pred) in consume’, the extra heaps tracked in consume’, and the heap assignments in the continuations from consume come together to implement heap chunk removal.
consume\(\sigma_1, \theta, Q\) = \sigma_2 := \sigma_1\{h, \pi := \text{consolidate}(\sigma_1, h, \sigma_1, \pi)\}

consume'\(\sigma_2, \sigma_2, \text{isImprecise}, \sigma_2, h_\tau, \theta, (\lambda \sigma_3, h'_\tau, h_1, \delta_1)\).

\(Q(\sigma_3\{h_\tau := h'_\tau, h := h_1\}, \delta_1))\)

consume\(\sigma_1, ? & \& \phi, Q\) = \sigma_2 := \sigma_1\{h, \pi := \text{consolidate}(\sigma_1, h, \sigma_1, \pi)\}

consume'\(\sigma_2, \sigma_2, \text{true}, \sigma_2, h_\tau, \theta, (\lambda \sigma_3, h'_\tau, h_1, \delta_1)\).

\(Q(\sigma_3\{\text{isImprecise} := \text{true}, h_\tau := 0, h := 0\}, \text{pair}(\text{unit}, \delta_1)))\)

\begin{align*}
\text{consume}'(\sigma, f_1, h_\tau, h, (e, t), Q) &= \text{res}, \overline{t} := \text{assert}(\sigma, \text{isImprecise}, \sigma, \pi, t) \\
\text{\quad } R' := \text{addcheck}(\sigma, R, e, \text{translate}(\sigma, \overline{t})) \\
\text{\quad } \text{res} \land Q(\sigma(\{R := R'\}, h_\tau, h, \text{unit}))
\end{align*}

\begin{align*}
\text{consume}'(\sigma_1, f_1, h_\tau, h, e, Q) &= \text{eval-e}(\sigma_1\{\text{isImprecise} := f_1\}, e, (\lambda \sigma_2, t) \cdot \\
\text{\quad } \text{consume}'(\sigma_2\{\text{isImprecise} := \sigma_1, \text{isImprecise}\}, f_1, h_\tau, h, (e, t), Q))
\end{align*}

\begin{align*}
\text{consume}'(\sigma_1, f_1, h_\tau, h, \text{acc}(e, f), Q) &= \text{eval-e}(\sigma_1\{\text{isImprecise} := f_1\}, e, (\lambda \sigma_2, t) \cdot \\
\text{\quad } \sigma_3 := \sigma_2\{\text{isImprecise} := \sigma_1, \text{isImprecise}\} \\
\text{\quad } \text{res}, \overline{t} := \text{assert}(\sigma_3, \text{isImprecise}, \sigma_3, \pi, t \neq \text{null}) \\
\text{\quad } \text{res} \land ( \\
\text{\quad } R' := \text{addcheck}(\sigma_3, R, \text{acc}(e, f), \text{translate}(\sigma_3, \overline{t})) \\
\text{\quad } (h_\tau, \delta_1, b_1) := \text{heap-rem-acc}(\sigma_3, \text{isImprecise}, h, \sigma_3, \pi, f(t)) \\
\text{\quad } \text{if } (\sigma_3, \text{isImprecise}) \text{ then } \\
\text{\quad } (h'_\tau, \delta_1, b_2) := \text{heap-rem-acc}(\sigma_3, \text{isImprecise}, h_\tau, \sigma_3, \pi, f(t)) \\
\text{\quad } \text{if } (b_1 = b_2 = \text{false}) \text{ then } \\
\text{\quad } R'' := \text{addcheck}(R', \text{acc}(e, f), \text{acc}(\text{translate}(\sigma_3, t), f)) \\
\text{\quad } \text{else } R'' := R' \\
\text{\quad } Q(\sigma_3\{R := R''\}, h'_\tau, h_1, (if (b_1) \text{ then } \delta_1 \text{ else } \delta_2)) \\
\text{\quad } \text{else if } (b_1) \text{ then } Q(\sigma_3\{R := R''\}, \sigma_3, h_\tau, h_1, \delta_1) \\
\text{\quad } \text{else failure()})
\end{align*}

\begin{itemize}
\item Handles imprecision
\item Handles run-time check generation and collection
\end{itemize}

Fig. 13. Select rules for symbolically consuming formulas

### 4.3.7 Symbolic execution of statements

The exec rules in Gradual Viper, which symbolically execute program statements, are largely unchanged from Viper. The only differences are 1) the rules now utilize versions of eval, produce, consume, and branch defined previously in this paper and 2) the rules track origins where appropriate. To provide an intuition, select rules for exec are given in Fig. 14; the full set of rules are listed in the Appendix §A.4. The exec function takes a symbolic state \(\sigma\), program statement \(stmt\), and continuation \(Q\). Then, exec symbolically executes \(stmt\) using \(\sigma\) to produce a potentially modified state \(\sigma'\), which is passed to the continuation.

Symbolic execution of field assignments first evaluates the right-hand side expression \(e\) to the symbolic value \(t\). Any field reads in \(e\) are either directly or optimistically verified using \(\sigma_1\). Then, the resulting state \(\sigma_2\) must establish write access to \(x.f\) in consume, i.e. \(\sigma_2 \Rightarrow \text{acc}(x.f)\). Calling consume also removes the field chunk for \(\text{acc}(x.f)\) from \(\sigma_2\) (if it is in there) resulting in \(\sigma_3\). Therefore,
\[
\text{exec}(\sigma_1, \_x.f := e, Q) = \begin{align*}
\text{eval}(\sigma_1, e, (\lambda \sigma_2, t . \text{consume}(\sigma_2, \_x.f), (\lambda \sigma_3, \_r . \text{produce}(\sigma_3, \text{acc}(x.f) \& \& x.f = t, \text{pair}(\text{fresh}, \text{unit}), (Q)))))) \\
\text{exec}(\sigma_1, \_ \tau := m(\tau), Q) = \begin{align*}
&\text{eval}(\sigma_1, \_ \tau, (\lambda \sigma_2, \_ \tau .) \\
&\frac{\text{R}'}{\text{R}'} := \sigma_2.R[\text{origin} := (\sigma_2, \tau := m(\tau), I)] \\
&\text{consume}((\sigma_2 | \_ \tau := \text{R}'), \text{methpre}[\text{methargs} \mapsto I], (\lambda \sigma_3,\_ \delta .)
\text{if (eqi-imp(methpre)) then}
\text{\sigma_3 := \sigma_3(isImprecise := \text{true}, h_1 := 0, h_2 := 0, y := havoc(\sigma_3, \_ \gamma))}
\text{else \sigma_4 := \sigma_4(y := havoc(\sigma_3, \_ \gamma))}
\text{produce}(\sigma_4, \text{methpost}[\text{methargs} \mapsto I][\text{methret} \mapsto z], \text{fresh},
(\lambda \sigma_5. \_Q(\sigma_5 | \text{R} := \sigma_5.R[\text{origin} := \text{none}])))}
\end{align*}
\]

Fig. 14. Select rules for symbolically executing program statements

the call to produce can safely add a fresh field chunk for \text{acc}(x.f) alongside \text{x.f} = t to \sigma_3 before it is passed to the continuation Q. Under the hood, run-time checks are collected and passed to Q.

The method call rule evaluates the arguments \_\tau to symbolic values \_I while making sure the origin is set properly for check and branch condition insertion, havocs existing assumptions about the variables being assigned to, produces knowledge from the postcondition, and finally continues after resetting the origin to none. An in-depth explanation is in the Appendix, along with the other exec rules and eqi-imp definition.

Note that while Gradual Viper treats predicates iso-recursively in all other cases, it makes an exception when consuming preconditions at method calls (and loop invariants before entering loops), which can be seen in the if-then in the method call rule (Fig. 14). If Gradual Viper determines the precondition (invariant) is eqi-recursively imprecise, then it will conservatively remove all the heap chunks from both symbolic heaps and make the state imprecise after the consume. This exception ensures the static verification semantics in Gradual Viper lines up with the equi-recursive, dynamic verification semantics encoded by GVC0 (describe in §4.4) such that Gradual C0 is sound. Interestingly, Wise et al. [2020]'s gradual verifier does not need this special case, because it does not optimize run-time checks with statically available information. Once optimization is introduced, the semantics across the two systems need to be more tightly integrated to ensure soundness. Zimmerman et al. [2024] alerted us to this issue and proposed the aforementioned solution.

4.3.8 Valid Gradual Viper programs. Finally, putting everything together, a Gradual Viper program is checked by examining each of its method and predicate definitions to ensure they are well-formed (formally defined in Appendix Fig. 31). The formal definitions are given in Fig. 15, and a more detailed description of the rules is given in the Appendix, §A.5. Intuitively, for each method, we define symbolic values for the method arguments, and then create an initial symbolic state by calling the produce function on the method precondition\(^7\). We then call the exec function on the method body, which symbolically executes the body and ensures that all operations are valid based on that precondition. Finally, we invoke the consume function on the final symbolic state and the postcondition, verifying that the former implies the latter. Throughout these operations a set of run-time checks is built up, which (along with success or failure) is the ultimate result of gradual verification.

\(^7\)Note, produce is part of well-formed.
4.4 Dynamic Verification: Encoding Run-time Checks into C0 Source Code

After static verification, Gradual Viper returns a collection of run-time checks \( \mathcal{R} \) that are required for soundness to GVC0. Then, GVC0 creates a C0 program from the run-time checks in \( \mathcal{R} \) and the original C0 program by encoding the checks in C0 source code. The C0 program is sent to the C0 compiler to be compiled, executed, and thus dynamically verified. We chose to encode the run-time checks directly in source code to avoid complexities from augmenting the C0 compiler with support for dynamic verification. Further, since C0 is a simple imperative language, any more expressive language should be able to encode the checks far more easily. That is, we hope this work serves as a guide to the developers of Gradual Viper frontends for other languages on how to implement efficient dynamic verification for gradual verification—especially, when modifying the compiler for their language is difficult. The rest of this section illustrates GVC0’s encoding of run-time checks into C0 source code via example. We also highlight design points in the encoding that minimize run-time overhead of the checks during execution.

Now, consider the C0 program in Fig. 16 that implements a method for inserting a new node at the end of a list, called insertLastWrapper. Note, when passed a non-empty list, insertLastWrapper calls insertLast from Fig. 1 to perform insertion (line 17). Here, insertLast is gradually verified with the simpler and fully specified (precise) acyclic predicate given on lines 1-4 in Fig. 16. For our purposes, we only need to know that insertLast’s precondition is \( ? \&\&\) acyclic(list) \&\& list != NULL and its postcondition is acyclic(\result) \&\& \result != NULL. The insertLastWrapper method is also gradually specified: its precondition is \( ? \) (line 7)—requiring unknown information—and its postcondition is acyclic(\result) (line 8)—ensuring the list after insertion is acyclic. Fig. 16 also contains run-time checks generated by Gradual Viper for insertLastWrapper, as highlighted in blue. The first check (lines 15-16) ensures the list 1 sent to insertLast (line 17) is acyclic, and is only required when 1 is non-empty (non-null). The second check (lines 20-21) ensures the list returned from insertLastWrapper is acyclic, and is only required when insertLastWrapper’s parameter 1 is empty (null). These checks are not executable by the C0 compiler; therefore, GVC0 takes the program and checks in Fig. 16 and returns the executable program in Fig. 17. That is, GVC0 encodes branch conditions (lines 15 and 20), predicates (lines 16 and 21), accessibility predicates (\texttt{acc} (1->val) and \texttt{acc}(1->next) in acyclic’s body, lines 1-4), and separating conjunctions (also in acyclic’s body) from Gradual Viper into C0 source code. We discuss the aforementioned encodings in sections §4.4.1, §4.4.2, and §4.4.3 respectively. While not in the insertLastWrapper example, GVC0 translates checks of simple logical expressions into C0 assertions: e.g. assert(\( y >= 0 \)).

4.4.1 Encoding branch conditions. Run-time checks contain branch conditions that denote the execution path a check is required on. For example, in Fig. 16 acyclic(\result) should only be

\begin{align*}
\text{verify(method } m(x:T) \text{ returns } (y:T)) &= \text{well-formed}(\sigma_0[y := \sigma_0,y[x \mapsto \text{fresh}] [y \mapsto \text{fresh}]), \text{meth}_{\text{pre}}, \text{fresh}, (\lambda \sigma_1 . \\
& \quad \text{well-formed}(\sigma_1[\text{isImprecise} := \text{false}, h_1 := \emptyset, h := \emptyset], \text{meth}_{\text{post}}, \\
& \quad \quad \text{fresh}, (\lambda \ _ . \ success()))) \\
& \quad \land \\
& \quad \text{exec}(\sigma_1, \text{meth}_{\text{body}}, (\lambda \sigma_2 . \\
& \quad \quad \text{consume}(\sigma_2, \text{meth}_{\text{post}}, (\lambda \sigma_3, \ _ . \\
& \quad \quad \quad \_ := \mathcal{R} \cup \sigma_3, \mathcal{Rcs} ; \text{success()}))))))) \\
\text{verify(predicate } p(x:T)) &= \text{well-formed}(\sigma_0[y := \sigma_0,y[x \mapsto \text{fresh}]], \text{pred}_{\text{body}}, \text{fresh}, (\lambda \ _ . \ success()))) \\
\end{align*}

Fig. 15. Rules defining a valid Gradual Viper program.
/*@ predicate acyclic ( Node * l) =

l == NULL ? true :
acc(l->val) && acc(l->next) &&
acyclic(l->next) ;@*/

Node * insertLastWrapper ( Node * l, int val )
//@ requires ?;
//@ ensures acyclic(\result );
{
if (l == NULL ) {
  l = alloc ( struct Node );
  l-> val = val ;
  l-> next = NULL ;
} else {
  l = insertLast (l, val );
  if (l == NULL ) {
    l = alloc ( struct Node );
    l-> id = addStructAcc(_ownedFields, 2);
    l->val = val ; l->next = NULL ;
  } else {
    l = insertLast (l,val , _ownedFields );
  }
}
return l;

/* Run-time checks from Gradual Viper */

Node* insertLastWrapper(Node* l, int val ,
  OwnedFields* _ownedFields )
{
  bool _cond_1 = l == NULL;
  if (l == NULL ) {
    l = alloc ( struct Node);
    l->id = addStructAcc(_ownedFields, 2);
    l->val = val ; l->next = NULL ;
  } else {
    if (!_cond_1) { acyclic(l, _ownedFields); }
  }
  OwnedFields* _tempFields =
  initOwnedFields(_ownedFields->instCntr);
  sep_acyclic(l, _tempFields);
  l = insertLast (l,val , _ownedFields );
  if (_cond_1) { acyclic(l, _ownedFields); }
  OwnedFields* _tempFields1 =
  initOwnedFields(_ownedFields->instCntr);
  sep_acyclic(l, _tempFields1);
  return l;
}

/* Run-time checks from GVC0 */

Fig. 16. Original insertLastWrapper program with run-time checks from Gradual Viper

Fig. 17. GVC0 generated insertLastWrapper program with run-time checks

cHECKED at lines 20-21 when l == NULL, as indicated by the branch condition (none,l == NULL,l ==
NULL). Therefore, GVC0 first encodes the condition l == NULL into C0 code. In general, conditions
are encoded as logical expressions in C0 and assigned to fresh boolean variables at the program
point where they originated—we call this versioning. Then, the boolean variable is used in checks
in place of the condition. For example, the origin and location pair (none,l == NULL) tells GVC0
that l == NULL must be evaluated at the program point in insertLastWrapper containing the 1 ==
NULL AST element. As a result, in Fig. 17 a boolean variable _cond_1 is introduced on line 4 to hold
the value of l == NULL. The condition variable _cond_1 is then used in the C0 run-time check for
acyclic(\result ) later in the program (line 16). To reduce run-time overhead, _cond_1 is also used
in the check for acyclic(l) on line 10, which relies on the same branch point (none,l == NULL).
Further, while not demonstrated here, GVC0 supports short-circuit evaluation of conditions on the
same execution path to reduce run-time overhead. Finally, note, GVC0 carefully places versioning
code after any run-time checks encoded at the program point where the condition originated to
ensure this new assignment code (not verified by Gradual Viper) is framed, i.e. correct.

4.4.2 Encoding predicates. Now that GVC0 has versioned the branch conditions in Fig. 16 into
variables, GVC0 can use the variables to develop C0 run-time checks. The Gradual Viper check
((none,l == NULL,¬(l == NULL)), (l=insertLast(l, val),acyclic(l),acyclic(l))) is translated
into if (!_cond_1) { acyclic(l, _ownedFields); } on line 10 in Fig. 17. GVC0 places this C0 check
according to the origin, location pair ((l=insertLast(l, val),acyclic(l)), which points to the
program point just before the call to insertLast on line 14. The branch condition becomes the
if statement with condition !_cond_1 (§4.4.1), and acyclic(l) is turned into the C0 function call
acyclic(l, _ownedFields). The acyclic function implements acyclic’s predicate body as C0 code: it
asserts true for empty lists and recursively verifies accessibility predicates (using _ownedFields) for
nodes in non-empty lists. That is, predicates are encoded and treated equi-recursively by GVC0.
For efficiency, separation of list nodes is encoded separately on lines 11-13. We discuss the dynamic
verification of accessibility predicates and the separating conjunction in C0 code next (§4.4.3). Finally, a similar C0 check is created for acyclic(result) on lines 16-19.

### 4.4.3 Encoding accessibility predicates and separating conjunctions

GVC0 implements run-time tracking of owned heap locations in C0 programs to verify accessibility predicates and uses of the separating conjunction.

**Encoding owned fields in C0 source code.** An owned field is a tuple \((id, field)\) where \(id\) is an integer identifying a struct instance (object in C0) and \(field\) is an integer indexing a field in the struct. The OwnedFields struct, which is implemented as a dynamic hash table to improve check performance, contains currently owned fields. That is, hashed object identifiers \((id)\) and then field identifiers \((field)\) are used to index into OwnedFields where a boolean that determines whether or not the field is currently owned is stored. Since objects are tracked with integers, all struct definitions in a C0 program are modified to contain an additional \_id field.

**Semantics of tracking owned fields (inspired by Wise et al. [2020]).** At the entry point to a C0 program (e.g. main), an empty OwnedFields struct called _ownedFields is allocated and initialized. This is not shown in Fig. 17. Then, when a new struct instance is created—such as allocating a new node on line 6 in Fig. 17—the \_id field is initialized with the value of a global integer instCntr that uniquely identifies the instance. The call to library function addStructAcc on line 7 performs this functionality and then increments instCntr. It also adds all fields in the struct instance (e.g. 1->val:(1->_id,0), 1->next:(1->_id,1), and 1->_id:(1->_id,2)) to _ownedFields and marks them as owned. The only other times _ownedFields can change are at method/function calls. Methods, like insertLast and insertLastWrapper, may add or drop owned fields during their executions. They may also contain run-time checks, such as the one for acyclic(1) on line 10, that need owned fields for verification. So, GVC0 adds an additional parameter to the their declarations (e.g. line 2, Fig. 17) to accept, initialize, and then modify _ownedFields in their contexts. A callee’s pre- and postcondition controls what owned fields are passed to and from the callee via this new parameter. When a method’s precondition is imprecise\(^8\), then any caller will pass all of its owned fields to the method, as on line 14 for the call to insertLast. After execution, the callee method returns all of its owned fields to the caller. When a method’s precondition is precise, then any caller only passes its owned fields specified by the precondition to the method. If the method’s postcondition is precise, then after execution the callee method returns all of its owned fields as before; otherwise, only the owned fields specified by the postcondition are returned. Finally, as an optimization, in precisely specified methods (no external—pre- and postconditions—or internal—loop invariants, unfolds, folds, etc.—specifications contain imprecision and no run-time checks are required), GVC0 does not implement any _ownedFields tracking. In this case, GVC0 uses the callee’s pre- and postcondition to modularly update _ownedFields in the caller.

**Verifying accessibility predicates and separating conjunctions with owned fields.** Now, _ownedFields tracking is used to verify accessibility predicates and uses of the separating conjunction. Run-time checks for accessibility predicates are turned into assertions that ensure the presence of their heap location in _ownedFields. For example, acc(1->val) looks like assertAcc(_ownedFields,1->_id,θ); in C0 code, where 0 is the index for val in the Node struct. The assertAcc library function indexes into _ownedFields using 1->_id and 0 and ensures a boolean entry is in there and is true; otherwise, assertAcc throws an error. Wherever GVC0 must check separation of heap locations (as indicated in run-time checks from Gradual Viper via a flag\(^9\)),—such as for the nodes in list 1 at lines 10-13—it

\(^8\)Here, a formula is also imprecise if it contains predicates that expose \(\exists\) when fully unrolled—an equi-recursive treatment.

\(^9\)Note, this flag is not formalized in this paper for simplicity.
creates (with the library method initOwnedFields) an auxiliary data structure _tempFields of type OwnedFields. We check that heap cells are disjoint by adding them one at a time to _tempFields and failing if the cell is already there. GVC0 generates a sep_X method for each predicate X to actually perform the separation check; and when done, discards _tempFields, as its purpose was only to check separation. Similar checks are created for the acyclic(result) check on lines 16-19.

5 EMPIRICAL EVALUATION

The seminal work on gradual typing [Siek and Taha 2006] selectively inserts run-time casts in support of optimistic static checking: for instance, whenever a function application is deemed well-typed only because of imprecision—such as passing an argument of the unknown type to a function that expects an integer—the type-directed cast insertion procedure inserts a run-time check. But if the application is definitely well-typed, no cast is inserted. This approach ensures that a fully-precise program does not incur any overhead related to run-time type checking. While it is tempting to assume that more precision necessarily results in better performance, the reality has been shown to be more subtle: both the nature of the inserted checks (such as higher-order function wrappers) as well as when/how often they are executed is of utmost importance [Muehlboeck and Tate 2017; Takikawa et al. 2016], and anticipating the performance impact of precision is challenging [Campora et al. 2018].

The performance of gradual verification has never been studied until now, due to the lack of a working gradual verifier. Here, we explore the relation between minimizing dynamic check insertion with statically available information and observed run-time performance in gradual verification with Gradual C0. Specifically, we explore the performance characteristics of Gradual C0 for thousands of partial specifications generated from four data structures, as inspired by Takikawa et al. [2016]'s work in gradual typing. In particular, we observe how adding or removing individual atomic formulas and ? within a specification impacts the degree of static and dynamic verification and, as a result, the run-time overhead of the program. Additionally, we compare the run-time performance of Gradual C0 to a fully dynamic approach, as readily available in C0. The aforementioned ideas are captured in the following research questions:

RQ1: As specifications are made more precise, can more verification conditions be eliminated statically?
RQ2: Does gradual verification result in less run-time overhead than a fully dynamic approach?
RQ3: Are there particular types of specification elements that have significant impact in run-time overhead, and can high overhead be avoided?

5.1 Creating Performance Lattices

We define a complete specification as being statically verifiable when all ?s are removed, and then a partial specification as a subset of formulas from a complete specification that are joined with ?. Like Takikawa et al. [2016], we model the gradual verification process as a series of steps from an unspecified program to a statically verifiable specification where, at each step, an element is added to the current, partial specification. An element is an atomic conjunct (excluding boolean primitives) in any type of method contract, assertion, or loop invariant. We form a lattice of partial specifications by varying which elements of the complete specification are included. We also similarly vary the presence of ? in formulas that are complete—contain the same elements as their counterparts in the statically verifiable specification—and have related fold and unfold statements in the partial specification. Otherwise, ? is always added to incomplete formulas. This strategy creates lattices where the bottom entry is an empty specification containing only ?s and the top entry is a statically verifiable specification. A path through a lattice is the set of specifications
Table 1. Description of benchmark examples. For each example, the table shows the complexity of the test program without verification, the number of sampled partial specifications, and the distribution of specification elements for the complete specification. Element counts are formatted as "Accessibility Predicate/ Predicate Instance/ Boolean Expression/ Imprecision"

created by appending $n$ elements or removing $\rho$s one at a time from the bottom to the top of the lattice. The large array of partial specifications created in each lattice closely approximates the positive specifications supported by the gradual guarantee [Wise et al. 2020], which are less precise variants of successfully verified programs. For reference, we give a more formal statement of the gradual guarantee:

Let $p_1$ and $p_2$ be Gradual C0 programs where $p_1 \sqsubseteq p_2$ (i.e. the formulas in $p_1$ are more precise than those in $p_2$). If $p_1$ statically verifies, then $p_2$ statically verifies. Additionally, $p_2$ must execute at least as far as $p_1$ executes at run time.

Now, to illustrate the aforementioned approach, consider the following loop invariant:

```c++
//@ loop_invariant sortedSeg(list, curr, curr->val) && curr->val <= val;
```

The invariant is made of two elements: the `sortedSeg` predicate instance and the boolean expression `curr->val <= val;`. The lattice generated for a program with this invariant has five unique specifications: four contain a combination of the two elements joined with $\land$, and the fifth is the complete invariant above.

5.2 Data Structures

To apply this methodology, we implemented and fully specified four recursive heap data structures with Gradual C0: binary search tree, sorted linked list, composite tree, and AVL tree. We chose these data structures because complete static specifications exist for them in prior work and they are interesting use cases for gradual verification. Linked list is implemented with a while loop rather than recursion. Binary search tree is a more complex data structure with a more complex property (BST property) than a linked list and uses recursion. Composite tree implements a structure where modifications do not have to start at the root, but can be applied directly to any node in the tree. Its invariant also applies to any node in the tree. Finally, AVL tree implements the most complex invariant (the balanced property) and data structure with many interdependent functions and predicates related to tree rotations. Each data structure has a test program that contains its implementation and a main function that adds elements to the structure based on a workload parameter $\omega$. We design the test programs to incur as little run-time overhead as possible outside of structure size and run-time checks. For each example and corresponding test program, Table 1 displays the distribution of elements in the complete specification, as well as the run-time complexity of the test program and the number of unique partial specifications generated by our benchmarking tool.

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Binary Search Tree</td>
<td>$O(n \log n)$</td>
<td>3473</td>
<td>44</td>
<td>23</td>
<td>0/20/21/24</td>
<td>6/22/6/24</td>
<td>6/6/7/4</td>
<td>0/2/4/2</td>
</tr>
<tr>
<td>Linked List</td>
<td>$O(n)$</td>
<td>1745</td>
<td>17</td>
<td>10</td>
<td>4/6/15/5</td>
<td>4/5/6/3</td>
<td>4/3/4/3</td>
<td>4/3/5/2</td>
</tr>
<tr>
<td>Composite</td>
<td>$O(n \log n)$</td>
<td>2577</td>
<td>28</td>
<td>15</td>
<td>0/10/2/12</td>
<td>6/11/1/12</td>
<td>32/9/17/3</td>
<td>0/3/2/3</td>
</tr>
<tr>
<td>AVL</td>
<td>$O(n \log n)$</td>
<td>3057</td>
<td>25</td>
<td>14</td>
<td>3/4/5/9</td>
<td>3/6/9/9</td>
<td>25/8/21/3</td>
<td>1/1/2/1</td>
</tr>
</tbody>
</table>

### Binary Search Tree (BST)

The implementation of the binary search tree is typical; each node contains a value and pointers to left and right nodes. We statically specify memory safety and preservation of the binary search tree property—that is, any node’s value is greater than any value in its left subtree and less than any value in its right subtree. The test program creates a root node with value $\omega$ and sequentially adds and removes a set of $\omega$ values in the range $[0, 2\omega]$. Note that values are removed in the same order they were added.
**Linked List.** We implement a linked list with insertion similar to the one given in Fig. 1. Insertion is statically specified for memory safety as well as preservation of list sortedness. Its test program creates a new list and inserts ω arbitrary elements.

**Composite.** The composite data structure is a binary tree where each node tracks the size of its subtree—this is verified by its specification along with memory safety. Its test program starts with a root node and builds a tree of size ω by randomly descending from the root until a node without a left or right subtree is reached. A new node is added in the empty position, and then traversal backtracks to the root.

**AVL Tree.** The implementation of AVL tree with insertion is standard except that the height of the left and right subtrees is stored in each node (instead of the overall height of the tree). This allows us to easily state the AVL balanced property—for every node in the tree the height difference between its left and right children is at most 1—without using functions or ghost variables, which Gradual C0 does not currently support. In addition to specifying the AVL balanced property for insertion, we also specify memory safety. The AVL test program starts with a root node and builds a tree of size ω by inserting randomly valued nodes into the tree using balanced insertion.

### 5.3 Experimental Setup

With upwards of 100 elements in the specifications for each data structure, it is combinatorially infeasible to fully explore every partial specification. Therefore, unlike Takikawa et al. [2016], we proceed by sampling a subset of partial specifications in a lattice, rather than executing them all. Specifically, we sample 16 unique paths through the lattice from randomized orderings of specification elements. We chose partial specifications along lattice paths to explore trends in migration from no specifications to complete specifications, which is how we imagine developers may use our tool. We also randomly sampled paths, rather than using another heuristic for selecting paths, to be prescriptive to users of Gradual C0. We wanted to find and recommend new specification patterns that users should apply or avoid depending on their performance. Every step is executed with three workloads chosen arbitrarily to ensure observable differences in timing. Each timing measurement is the median of 10 iterations. Programs were executed on four physical Intel Core i5-4250U 1.3GHz Cores with 16 GB of RAM.

We introduce two baseline verifiers to compare Gradual C0 against. The dynamic verifier transforms every specification into a run-time check and inserts accessibility predicate checks for field dereferences—thereby emulating a fully dynamic verifier. The framing verifier only performs the accessibility predicate checks, and therefore represents the minimal dynamic checks that must be performed in a language that checks ownership. We implement the baseline verifiers ourselves using Wise et al. [2020]’s dynamic semantics, which checks everything at run time, as a guide. Wise et al. [2020]’s work is the only work that we are aware of that handles run-time checking of both ownership and recursive predicates.

### 5.4 Evaluation

Fig. 18 shows how the total number of verification conditions (proof obligations) changes as more of each benchmark is specified (green curve). The figure also similarly shows the number of verification conditions that are statically verified as each benchmark is specified (purple curve). From the green curve, we see that even when there are no specifications, there are verification conditions, e.g. before a field is accessed, the object reference must be non-null and the field must be

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10These framing checks could fail, for example, if some function lower in the call stack owns data that is accessed by the currently-executing function.
Mean Verification Conditions

![Graph](image)

**Fig. 18.** For each example, the average quantity of verification conditions and the subset that were eliminated statically at each level of specification completeness across all paths sampled. Shading indicates the standard deviation.

Mean Execution Time Over All Paths

![Graph](image)

**Fig. 19.** The mean time elapsed at each step over the 16 paths sampled. Shading indicates the confidence interval of the mean for each verification type.

owned. Some of these verification conditions can be verified statically as illustrated by the purple curve. As more of a benchmark is specified, there are more verification conditions (green curve); but also, more of these verification conditions are discharged statically and do not have to be checked dynamically (purple curve). Towards the right end of the plots, the two curves converge until they meet when all the verification conditions are discharged statically. As a result, the answer to **RQ1** is yes. Note, the number of verification conditions does decrease when enough of the benchmark is specified. This is due to being able to prune symbolic execution paths with new static information.

The plots in Fig. 19 display the run-time performance (in red) of dynamically checking the verification conditions from Fig. 18. The plots also show how the run-time performance of the dynamic verifier (in green) and framing verifier (in purple) change as more of each benchmark is specified. The green lines show that as more properties are specified, the cost of run-time verification increases. With Gradual C0, some of these properties can be checked statically; therefore, the run-time cost of gradual verification, shown in red, starts equivalent but eventually ends up significantly lower than the cost of pure run-time verification.

Notably, the purple lines are significantly lower than the red and greens ones until they exhibit a dramatic increase starting at around 80% specified all the way to 100%. Eventually (after about 95% specified), the purple lines end above the red ones (but below the green ones) where running time is orders of magnitude higher than at the start of the incline. The framing verifier (in purple) only checks that heap accesses are safe—*i.e.* they are owned and their receivers are non-null. So unsurprisingly, the dynamic and gradual verifiers, which check more properties like heap
Table 2. Summary statistics for the performance of each example over 16 paths at selected workloads ($\omega$), comparing gradual verification (GV) against dynamic verification (DV). The grouped column "% in $\Delta t$, GV vs. DV." displays summary statistics for the percent decrease in time elapsed for each step when using GV versus DV. The column "% Steps GV<DV for Paths DV<GV" shows the distribution of steps that performed best under GV that were part of paths containing steps that performed better under DV. The final column shows the percentage of paths in which every step performed better under GV.

<table>
<thead>
<tr>
<th>Example</th>
<th>$\omega$</th>
<th>% $\Delta t$, GV vs. DV</th>
<th>% Steps GV&lt;DV for Paths DV&lt;GV</th>
<th>% Paths GV&lt;DV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>St. Dev.</td>
<td>Max</td>
<td>Min</td>
</tr>
<tr>
<td>AVL</td>
<td>32</td>
<td>-14.4</td>
<td>29.4</td>
<td>170.0</td>
</tr>
<tr>
<td></td>
<td>64</td>
<td>-16.1</td>
<td>50.5</td>
<td>256.3</td>
</tr>
<tr>
<td></td>
<td>128</td>
<td>-11.5</td>
<td>74.0</td>
<td>384.7</td>
</tr>
<tr>
<td>BST</td>
<td>32</td>
<td>-24.7</td>
<td>35.0</td>
<td>144.5</td>
</tr>
<tr>
<td></td>
<td>64</td>
<td>-23.9</td>
<td>46.8</td>
<td>301.3</td>
</tr>
<tr>
<td></td>
<td>128</td>
<td>-21.7</td>
<td>55.6</td>
<td>436.1</td>
</tr>
<tr>
<td>Linked List</td>
<td>32</td>
<td>-18.0</td>
<td>27.7</td>
<td>138.9</td>
</tr>
<tr>
<td></td>
<td>64</td>
<td>-21.0</td>
<td>48.6</td>
<td>359.0</td>
</tr>
<tr>
<td></td>
<td>128</td>
<td>-20.7</td>
<td>59.7</td>
<td>603.3</td>
</tr>
<tr>
<td>Composite</td>
<td>32</td>
<td>-34.4</td>
<td>40.1</td>
<td>141.7</td>
</tr>
<tr>
<td></td>
<td>64</td>
<td>-33.1</td>
<td>50.1</td>
<td>258.6</td>
</tr>
<tr>
<td></td>
<td>128</td>
<td>-30.1</td>
<td>65.9</td>
<td>419.0</td>
</tr>
</tbody>
</table>

The dramatic increase in the framing verifier’s run-time performance is caused by the owned fields passing strategy employed at method boundaries (described in §4.4.3) to verify memory safety at run time. To respect precondition abstractions, only owned fields specified by a callee’s precondition are passed by the caller to the callee when the precondition is precise. Similarly, when a callee’s postcondition is precise, then only the owned fields specified by the postcondition are passed back to the caller. Computing owned fields from precise contracts is costly; and even more-so for contracts containing recursive predicates like in our benchmarks. Further, our benchmarks call such methods frequently during execution. As a result, execution time increases significantly at each path step where one of the aforementioned methods gets a precise pre or postcondition from ? removal. This, of course, happens more frequently as more of a benchmark is specified. At 100% specified every method contract is precise, and so the owned fields passing strategy is used at every method call and return leading to the highest run-time costs for the framing verifier. In contrast, Gradual C0 checks fully-specified methods completely statically and does not use the owned field passing strategy for calls to these methods. As a result, looking at the red lines, Gradual C0 is not heavily affected by this phenomena—we see slight increases starting at 90% specified but they are significantly less costly. Additionally, once a critical mass of specifications have been written, Gradual C0’s run-time verification cost decreases until reaching zero—which is the same as running the raw C0 version of the benchmark. If the spikes around 90% specified are too costly, production gradual verifiers can reduce them by employing more optimal permission passing strategies.

In general, according to the red lines, Gradual C0’s performance increases gradually as more proof obligations are specified but are not yet statically verified; and thus, must be checked at run time. When a critical mass of specifications are written that allows more and more of these proof obligations to be proven statically, run-time performance starts to decrease until reaching the spikes around 90% specified caused by owned fields passing. After the spikes, performance decreases to the benchmark’s raw baseline. This trend is consistent with speculations made in Wise et al. [2020]’s work, and also confirms that increasing precision in gradual verification does not always correspond with decreased run-time overhead from dynamic verification.

Table 2 displays summary statistics for Gradual C0’s performance on every sampled partial specification compared to the dynamic verification baseline. Depending on the workload and separation, nearly always have significantly higher run-time verification overhead than the framing verifier. Eventually, Gradual C0 outperforms the framing verifier when enough properties, including framing, are checked statically.
example, on average Gradual C0 reduces run-time overhead by 11.5-34.4% (Table 2, Column 3) compared to the dynamic verifier. Note, the speed-ups are consistent as $\omega$ increases: -14.4%, -24.7%, -18.0%, and -34.4% at the lowest $\omega$ values compared to -11.5%, -21.7%, -20.7%, and -30.1% at the largest. While Gradual C0 generally improves performance, there are some outliers in the data (Table 2, Column 5) where Gradual C0 is slower than dynamic verification by 138.9-603.3%. Fortunately, for lattice paths that produce these poor-performing specifications, gradual verification still outperforms dynamic verification (on average) for 74.2-85.8% (Table 2, Column 7) of all steps. Further, these outliers appear under 20% specified where the bookkeeping we insert to track conditionals, which is unoptimized and could be improved, and measurement error are the cause of such outcomes.

Fig. 19 displays the average run-time cost across all paths under each of our benchmarks and verifiers. In all the plots, for some early parts of the path the cost of Gradual C0 is comparable to or exceeds the cost of the dynamic verifier, but after 50% completion, static optimization kicks in and Gradual C0 begins to significantly outperform it. Further, Table 2 shows that on average Gradual C0 reduces run-time overhead by 11.5-34.4% compared to the dynamic verifier. Therefore, the answer to RQ2 is yes.

Fig. 20 captures the impact that different types of specification elements (accessibility predicates, predicates, and boolean expressions) have on Gradual C0’s run-time performance when specified in different locations. It also captures the impact removing $\exists$ from a formula has on performance. Elements that when added or $\exists$ that when removed from one step in a lattice path to another increase run-time overhead significantly (in the top 1%) are counted in the left sub-figure, and ones that decrease run-time overhead significantly (top 1%) are counted in the right sub-figure. The count for accessibility predicates is colored in green, predicates in purple, boolean expressions in yellow, and $\exists$ removal in red.

Adding recursive predicates to preconditions, postconditions, and predicate bodies is the most frequent cause (67.6%) of dramatic increases in run-time verification overhead during the specification process. When these predicates are added to preconditions and postconditions they create additional proof obligations for them in callers and callees (respectively) that are frequently checked at run time. Similarly, when they are added to predicate bodies any proof obligations for the enclosing predicate that are checked at run time become far more expensive. Fortunately, folding or unfolding a predicate can decrease run-time cost when doing so discharges such proof obligations statically.
(as seen in the right sub-figure). Therefore, users of Gradual C0 may consider specifying proofs of recursive predicates in frequently-executed code to significantly reduce checking costs.

Removing ? from preconditions, postconditions, and predicate bodies when the costly owned fields passing strategy is still required in corresponding methods is the second most frequent cause (27.9%) of increases in Gradual C0’s run-time overhead. This corresponds with the spikes at 90% specified in Fig. 19 for Gradual C0: removal of ? in the aforementioned locations leads to precise pre- and postconditions that trigger the use of this costly strategy. Eventually, a critical mass of specifications are written so that when ?s are removed further this costly strategy is no longer necessary (i.e. when callee methods are full statically verified) and so run-time performance improves dramatically—the downward trends seen prior to full static specification in Fig. 19. This is reflected in the right sub-figure in Fig. 20, where removing ? from preconditions, postconditions, and predicate bodies is the most frequent cause (68.2%) of significant decreases in run-time overhead. This suggests a strategy for avoiding high checking costs: specify frequently-executed code in critical-mass chunks that are fully statically verifiable, leaving boundaries between statically and dynamically verified code in places that are executed less frequently.

Overall, the answer to RQ3 is yes; we have identified some key contributors to run-time overhead, whose optimization is a promising direction for future work, and we have also identified strategies for minimizing run time overhead in practice.

Finally, all of the partial specification evaluated in our study were successfully verified by Gradual C0. Since they originated from complete and correct specifications on code, we can conclude Gradual C0 adheres to the gradual guarantee for these partial specifications and likely adheres to the gradual guarantee for common use cases of Gradual C0.

**Threats to Validity.** Our test programs were executed on multiple devices, each with the same CPU and memory configuration. However, we did not otherwise control for differences in performance between devices. While the test programs we used are of sufficient complexity to demonstrate interesting empirical trends, they are not representative of all software. Further, the baseline we used for dynamic verification is entirely unoptimized as we naively insert a check for each written element of a specification. Finally, due to computational constraints, only a small subset of over \(2^{100}\) possible imprecise specifications were sampled, and we did not use a formal criteria to choose our workload values. As such, while our results reveal interesting trends, including significant performance improvements by Gradual C0 over dynamic verification, more work is needed to validate the robustness of those trends.

### 5.5 Qualitative Experience with AVL Tree

Notably, it was our experience that the incrementality of gradual verification was very helpful for developing a complete specification of the AVL tree example. In particular, a run-time verification error from a partial specification helped us realize the contract for the rotateRight helper function was not general enough. We fully specified rotateRight and proved it correct. However, insert’s pre- and postconditions were left as ?, and so static verification could not show us that the contract proved for rotateRight was insufficiently general. Nevertheless, we ran the program; gradual verification inserted run-time checks, and the precondition for rotateRight failed. This early notification allowed us to identify the problem with the specification and fix it immediately. Otherwise, we would have had to get deep into the static verification of insert—a complicated function, 50 lines long, with lots of tricky logic and invariants—before discovering the error, and a lot of verification work built on the faulty specification would have had to be redone. Interestingly, it is conventional wisdom that one of the benefits of static checking is that you get feedback early, when it is easier to correct mistakes. Here, we encountered a scenario where gradual verification had a similar benefit.
over static verification! We found an error (in a specification) earlier than we would have otherwise, presumably saving time.

6 RELATED WORK

Much of the closely related work, particularly work on gradual verification [Bader et al. 2018; Wise et al. 2020; Zimmerman et al. 2024], gradual typing [Herman et al. 2010; Siek and Taha 2006; Siek et al. 2015; Takikawa et al. 2016], and static verification [Müller et al. 2016; Parkinson and Bierman 2005; Reynolds 2002; Smans et al. 2009], has been discussed throughout the paper. Note, this paper and Zimmerman et al. [2024]’s paper have distinct contributions. Our work includes the original development of symbolic execution based gradual verification and describes the first implementation of the same, as well as related empirical results. Zimmerman et al. [2024]’s paper proves that the approach is sound. Now, we discuss additional related work.

Gradual typing. Additional related work in gradual typing includes richer type systems such as gradual refinement types [Lehmann and Tanter 2017] and gradual dependent types [Eremondi et al. 2019; Lennon-Bertrand et al. 2022]. These systems focus on pure functional programming, while Gradual C0 targets imperative programs. There is an extensive body of work on optimizing run-time checks in gradual type systems. Muehlboeck and Tate [2017] show that in languages with nominal type systems, such as Java, gradual typing does not exhibit the usual slowdowns induced by structural types. Feltey et al. [2018] reduce run-time overhead from redundant contract checking by contract wrappers. They eliminate unnecessary contract checking by determining—across multiple contract checking boundaries for some datatype or function call—whether some of the contracts being checked imply others. While the results in §5 are promising, we may be able to draw from the extensive body of work in gradual typing to achieve further performance gains.

Static verification. Work in formal verification contains approaches that try to reduce the specification burden of users—a goal of Gradual C0. Furia and Meyer [2010] infer loop invariants with heuristics that weaken postconditions into invariants. When that approach fails, verification also fails because invariants are missing. Similarly, several tools (Smallfoot [Berdine et al. 2006], jStar [Distefano and Parkinson J 2008], and Chalice [Leino et al. 2009]) use heuristics to infer fold and unfold statements for verification. In contrast, Gradual C0 does not fail solely because invariants, folds, or unfolds are missing; imprecision begets optimism. However, Gradual C0 may benefit from similar heuristic approaches by leveraging additional static information to further reduce run-time overhead.

Abductive reasoning (abductive inference) tries to find an explanatory hypothesis for a desired outcome [Dillig et al. 2012]. In static verification, the desired outcome is a proof obligation (O). facts (F) are invariants derived from the program and specifications using some analysis, and the explanatory hypothesis (E) are invariants that do not contradict the derived facts (SAT(F ∧ E)) and are required to discharge the proof obligation (F ∧ E ⊨ O). Ideally, F should be sufficient to discharge O, but missing or insufficient specifications often results in F being too weak to prove O leading to false positives (alarms) in tools. So, work in applying abductive reasoning to static verification [Blackshear and Lahiri 2013; Calcagno et al. 2009; Chandra et al. 2009; Das et al. 2015; Dillig et al. 2012] aims to compute E in order to prioritize—with minimal human intervention—verification failures caused by bugs in a program and de-emphasize false positives (alarms) caused by missing or incomplete specifications. In angelic verification [Blackshear and Lahiri 2013; Das et al. 2015] and Calcagno et al. [2009]’s work, entire specifications, such as preconditions, postconditions, and loop invariants are generated as explanatory hypotheses. Dillig et al. [2012] instead compute smaller, intermediate formulas as explanatory hypotheses.
Similar to prior abductive reasoning work [Blackshear and Lahiri 2013; Calcagno et al. 2009; Das et al. 2015; Dillig et al. 2012], Gradual C0’s static system reasons around missing or incomplete specifications to compute facts \( F \) as part of imprecise formulas \( \varphi \land F \). At proof obligations, we approximate the weakest formula that can replace \( \varphi \) in \( \varphi \land F \vdash O \) and SAT(\( \varphi \land F \)) successfully. So, like Dillig et al. [2012] we compute intermediate explanatory hypotheses rather than whole specifications like Blackshear and Lahiri [2013], Calcagno et al. [2009], and Das et al. [2015]. But, rather than relying on users to validate generated hypotheses [Blackshear and Lahiri 2013; Calcagno et al. 2009; Das et al. 2015; Dillig et al. 2012], we check their correctness at run time. This significantly simplifies their computation—since they do not need to be human readable and can statically mark code as unreachable—and allows Gradual C0 to be sound (prior abduction work is not).

**Dynamic verification.** Meyer [1988] introduced the Eiffel language, which automatically performs dynamic verification of pre- and postconditions and class invariants in first order logic. Nguyen et al. [2008] extended dynamic verification to support separation logic assertions. More recently, Agten et al. [2015] applied dynamic checking at the boundaries between statically verified and unverified code to guarantee that no assertion failures or invalid memory accesses occur at run time in any verified code. Their approach improved on Nguyen et al. [2008]’s approach in terms of performance by allowing unverified code to read arbitrary memory. Further, unlike Nguyen et al. [2008], Agten et al. [2015]’s approach only needs access to verified code rather than the entire codebase. As with Nguyen et al. [2008]’s work, Gradual C0 supports dynamic verification of ownership and first order logic. Gradual C0 additionally supports run-time checking of recursive predicates. Similarly to Agten et al. [2015], Gradual C0 applies dynamic checking at the boundaries between verified and unverified code to protect verified code. However, in Gradual C0 unverified code must be accessible to the verifier as it is gradually verified as well. Future work in gradual verification should incorporate insights from Agten et al. [2015]’s work to avoid requiring entire codebases for verification and to improve verification performance.

**Hybrid verification.** Another closely related work is soft contract verification [Nguyen et al. 2014], which verifies dynamic contracts statically where possible and dynamically where necessary by utilizing symbolic execution. This hybrid technique does not rely on a notion of precision, which is central to gradual approaches and their metatheory [Siek et al. 2015]. Nguyen et al. [2014] use symbolic execution results directly to discharge proof obligations where possible, while Gradual C0 strengthens symbolic execution results to discharge proof obligations adhering to the theory of imprecise formulas from Wise et al. [2020]. Further, Nguyen et al. [2014]’s work is targeted at dynamic functional languages, while our work focuses on imperative languages. We also build in memory safety as a default, while Nguyen et al. [2014] do not.

7 CONCLUSION

Gradual verification is a promising approach to supporting incrementality and enhance adoptability of program verification. Users can focus on specifying and verifying the most important properties and components of their systems and get immediate feedback about the consistency of their specifications and the correctness of their code. By relying on symbolic execution, Gradual C0 overcomes several limitations of prior work on gradual verification of heap-manipulating programs. The experimental results show that our approach can reduce overhead significantly compared to purely dynamic checking and confirms performance trends speculated in prior work. While more work remains to extend gradual verification (and Gradual C0) to the expressiveness of state-of-the-art static program verifiers, we believe the symbolic-execution approach to gradual verification shows promise to make verification more adoptable in software development practice.
REFERENCES


David Herman, Aaron Tomb, and Cormac Flanagan. 2010. Space-efficient gradual typing. 23, 2 (June 2010), 167–189.


Gradual C0


**A APPENDIX**

\[ \text{pc-add}(\pi, t) = \text{Let } (id, bc, pcs) :: \text{suffix match } \pi \]
\[ (id, bc, pcs \cup \{t\}) :: \text{suffix} \]
\[ \text{pc-push}(\pi, id, bc) = (id, bc, \emptyset) :: \pi \]
\[ \text{pc-all}(\pi) = \text{foldl}(\pi, \emptyset, (\lambda (id, bc, pcs) \cdot (\text{all}_{id}. \text{all}_{bc} \cup \{bc\} \cup pcs))) \]

Fig. 21. Path condition helper functions

\[ \text{eval-p}(\sigma, t, Q) = Q(\sigma, t) \]
\[ \text{eval-p}(\sigma, x, Q) = Q(\sigma, \sigma.y(x)) \]
\[ \text{eval-p}(\sigma_1, op(\bar{\tau}), Q) = \text{eval-p}(\sigma_1, \bar{\tau}, (\lambda \sigma_2. \text{All}. Q(\sigma_2, op'(T)))) \]
\[ \text{eval-p}(\sigma_1, e.f, Q) = \text{eval-p}(\sigma_1, e, (\lambda \sigma_2. \text{All}. t. \text{check}(\sigma_2, \pi, r = t)) \]
\[ \text{if } (\exists f(r; \delta) \in \sigma_2. h \cdot \text{check}(\sigma_2, \pi, r = t)) \text{ then} \]
\[ Q(\sigma_2, \delta) \]
\[ \text{else if } (\exists f(r; \delta) \in \sigma_2. h ? \cdot \text{check}(\sigma_2, \pi, r = t)) \text{ then} \]
\[ Q(\sigma_2, \delta) \]
\[ \text{else if } (\sigma_2.\text{isImprecise}) \text{ then} \]
\[ \text{if } (\sigma_2.R.\text{origin} = (_{\cdot}, \text{unfold acc}_{\cdot}, _{\cdot})) \text{ then} \]
\[ e_i := \text{translate}(\sigma_2, t) \]
\[ R' := \text{addcheck}(\sigma_2.R, e.f, \text{acc}(e.f)) \]
\[ \text{else} \]
\[ R' := \sigma_2.R \]
\[ \delta := \text{fresh} \]
\[ Q(\sigma_2, h_f := \sigma_2.h_f \cup f(t; \delta), \pi := \text{pc-add}(\sigma_2, \pi, \{t \neq \text{null}\}), R := R', \delta) \]
\[ \text{else failure}() \]

Fig. 22. Rules for symbolically executing expressions without introducing run-time checks (except for a special case for unfold)

In eval-p (Fig. 22), a special case (highlighted in blue) for unfold statements is added that creates run-time checks for field accesses in the unfolded predicate’s body. This case ensures soundness when introducing branch condition variables in C0 programs during run-time verification. In our implementation of Gradual C0, these checks are optimized further as they are only produced for branch conditions in the predicate body rather than for the whole body.
eval-c(σ, t, Q) = Q(σ, t)
eval-c(σ, x, Q) = Q(σ, σ.y(x))
eval-c(σ1, op(τ), Q) = eval-c(σ1, τ, (λ σ2, T . Q(σ2, op′(T))))
eval-c(σ1, e.f, Q) = eval-c(σ1, e, (λ σ2, t .
if (∃ f(r; δ) ∈ σ2.h . check(σ2.π, r = t)) then
Q(σ2, δ)
else if (∃ f(r; δ) ∈ σ2.h? . check(σ2.π, r = t)) then
Q(σ2, δ)
else if (σ2.isImprecise) then
res, _ := assert(σ2.isImprecise, σ2.π, t ≠ null)
e_{t} := translate(σ2, t)
R′ := addcheck(σ2.R, e.f, acc(e_{t}, f))
res ∧ Q(σ2{ R := R′ }, fresh)
else failure()
)

Fig. 23. Rules for symbolically executing expressions without modifying the optimistic heap and path condition

A.1 Diff and Translate

Algorithm 1 Generating minimal checks

1: function Diff(ϕ)
  2:   conjuncts ← CNF(ϕ)
  3:   ϕ′ ← ∅
  4:   for c ← conjuncts do
  5:     if !check(c) then
  6:       ϕ′ ← ϕ′ + c
  7:     end if
  8:   end for
  9:   return ϕ′
10: end function

Fig. 24. Algorithm for computing the diff between two symbolic values

The DIFF (Fig. 24) function finds a minimal run-time check from an optimistically asserted formula containing statically known information. It accomplishes this by first performing a standard transformation to conjunctive normal form (CNF) on the optimistically asserted formula, to extract the maximal number of top level conjuncts. It then attempts to call check() on each conjunct; it accumulates each conjunct for which the call does not succeed. The set of conjuncts which could not be statically discharged are returned as the final check.

The TRANSLATE (Fig. 25) function lifts symbolic values to concrete values. Most symbolic values are directly translated to their concrete counterparts via recursive descent; the exception is variables, whose concrete values must be reconstructed by searching the program state known by the verifier. This is done by retrieving the states of the symbolic store, which contains mappings from concrete variables to symbolic variables, and the heap, which contains field and predicate permissions. When TRANSLATE encounters a symbolic variable, it first retrieves all possible aliasing information from
Algorithm 2 Variable resolution procedure

1: function Translate-var(s,v)
2:     store ← ∅
3:     if s.oldStore then
4:         store ← s.oldStore
5:     else
6:         store ← s.store
7:     end if
8:     aliasList ← aliases(v,s.pathConditions ++s.heap ++s.optimisticHeap)
9:     heap ← s.heap ++s.optimisticHeap
10:    outputs ← ∅
11:    for v ← aliasList do
12:        if c ← store.lookup(v) then
13:            outputs ← outputs + c
14:        else
15:            if h ← heap.lookup(v)&&c ← store.lookup(h) then
16:                outputs ← outputs + c
17:            end if
18:        end if
19:    end for
20:    return selectLongest(outputs)
21: end function

Fig. 25. Translate’s procedure for resolving variables

Gradual Viper’s state. This includes all variables known to be equivalent to the translation target according to the path condition and the heap. If the translation target or one of its aliases exists as a value in the symbolic store, then the translator finds a key corresponding to it in the store and returns it. Note that multiple valid keys may exist for a particular symbolic variable, because Gradual Viper may have determined that multiple concrete values are equivalent at a particular program point. If the translation target is a field, then only the top level receiver (the variable on which fields are being accessed) or one of its aliases will exist in the store. The fields being accessed are resolved by mapping their corresponding heap entries, or any aliased heap entries, to a value in the symbolic store, and resolving the store entry as described. In particular contexts, Translate may be asked to translate a precondition for a method call, or a predicate body for an (un)fold statement. In these cases, an old store attached to the current symbolic state as described in 4.3.7 is retrieved, and its symbolic store and heap are used for translation. This causes variables in a precondition or predicate to be resolved to their concrete values at the call site, or site of unfolding. This enables run-time checks produced via translate to be straightforwardly emitted to the frontend. The portion of translate related to translating variables is shown in Fig. 25.

A.2 Symbolic production of formulas

The rules for produce are given in Fig. 11. Essentially, produce takes a formula and snapshot δ (mirroring the structure of the formula) and adds the information in the formula to the symbolic state, which is then returned to the continuation Q. An imprecise formula ?&&φ has its static part φ produced into the current state σ alongside second(δ). Note the snapshot δ for an imprecise formula
looks like \((unit, \text{second}(\delta))\) where \(unit\) is the snapshot for \(t\) and \(\text{second}(\delta)\) is the snapshot for \(\phi\). An imprecise formula also turns \(\sigma\) imprecise to produce the unknown information represented by \(\phi\). For example, if the state is represented by the formula \(\theta\), then this rule results in \(? \&\& \theta \&\& \phi\). A symbolic value \(t\) is produced into the path condition of the current state \(\sigma\). Also, the snapshot \(\delta\) for \(t\) must be \(unit\), so this fact is also stored in \(\sigma\)’s path condition. Then, \(\sigma\) is passed to \(Q\).

The produce rule for expression \(e\), first evaluates \(e\) to its symbolic value \(t\) using \(\text{eval-p}\). Then, \(t\) is produced into the path condition of the current state \(\sigma_2\) using the aforementioned symbolic value rule. Imprecision in the symbolic state can always provide accessibility predicates for fields also in the state. Therefore, when fields in \(e\) are added to an imprecise state, heap chunks for those fields do not have to already be in the state, e.g., the state \(? \&\& \text{true}\) becomes \(? \&\& \text{true} \&\& e\). This functionality is permitted by \(\text{eval-p}\). Similarly, an imprecise formula always provides accessibility predicates for fields in its static part, e.g., the state \(\text{true}\) and produced formula \(? \&\& e\) results in the state \(? \&\& \text{true} \&\& e\). The goal of produce is not to assert information in the state, but rather add information to the state. So we reduce run-time overhead by ensuring no run-time checks are produced by produce even for verifying field accesses.

The rules for producing field and predicate accessibility predicates into the state \(\sigma_1\) operate in a very similar manner. Thus, we will focus on the rule for fields only. The field \(e.f\) in \(\text{acc}(e,f)\) first has its receiver \(e\) evaluated to \(t\) by \(\text{eval-pc}\), resulting in \(\sigma_2\). Then, using the parameter \(\delta\) a fresh heap chunk \(f(t; \delta)\) is created and added to \(\sigma_2\)’s heap \(h\), which represents \(\text{acc}(e,f)\) in the state. Note, the disjoint union \(\omega\) ensures \(f(t; \delta)\) is not already in the heap before adding \(f(t; \delta)\) in there. If the chunk is in the heap, then verification will fail. Further, \(\text{acc}(e,f)\) implies \(e \neq \text{null}\) and so that fact is recorded in \(\sigma_2\)’s path condition as \(t \neq \text{null}\).

When the separating conjunction \(\phi_1 \&\& \phi_2\) is produced, \(\phi_1\) is first produced and then afterwards \(\phi_2\) is produced into the resulting symbolic state. Note that the snapshot \(\delta\) is split between the two formulas using \(\text{first}(\delta)\) and \(\text{second}(\delta)\). Finally, to produce a conditional, Gradual Viper branches on the symbolic value \(t\) for the condition \(e\) splitting execution along two different paths. Along one path only the true branch \(\phi_1\) is produced into the state, and along the other path only the false branch \(\phi_2\) is produced. Both paths follow the continuation to the end of its execution. More details about branching are provided next, as we describe Gradual Viper’s branch function.

The branch function in Fig. 12 is used to split the symbolic execution into two paths in a number of places in our algorithm: during the production or consumption of logical conditionals and during the execution of if statements. One path \((Q_t)\) is taken under the assumption that the parameter \(t\) is true, and the other \((Q_{\neg t})\) is taken under the assumption that \(t\) is false. For each path, a branch condition corresponding to the assumption made is added to \(\sigma.R\), as highlighted in blue. Additionally, paths may be pruned using check when Gradual Viper knows for certain a path is infeasible (the assumption about \(t\) would contradict the current path conditions). Now, normally, if either of the two paths fail verification, then branch marks verification as failed \((\lor \text{the results})\). This is still true when \(\sigma\) (the current state) is precise. However, when \(\sigma\) is imprecise, branch can be more permissive as highlighted in yellow. If verification fails on one of two paths only (one success, one failure), then branch returns success \((\lor \text{the results})\). In this case, a run-time check (highlighted in blue) is added to \(R\) to force run-time execution down the success path only. Of course, two failures result in failure and two successes result in success \((\lor \text{the results})\). No run-time checks are produced in these cases, as neither path can be soundly taken or both paths can be soundly taken at run time respectively. Note that Gradual Viper being flexible in the aforementioned way is critical to adhering to the gradual guarantee at branch points.

A.3 Symbolic consumption of formulas
We use the definition of consolidate we discuss later in this section. Therefore, in this case, the boolean flag is set to \( \sigma \) (is consume') ≠ \( \chi \). Residual heap chunks are instead represented by imprecision, e.g. handles imprecision.

The goals of consume are 3-fold: 1) given a symbolic state \( \sigma \) and formula \( \tilde{\phi} \) check whether \( \tilde{\phi} \) is established by \( \sigma \), i.e. \( \tilde{\phi}_\sigma \Rightarrow \tilde{\phi} \) where \( \tilde{\phi}_\sigma \) is the formula which represents the state \( \sigma \), 2) produce and collect run-time checks that are minimally sufficient for \( \sigma \) to establish \( \tilde{\phi} \) soundly, and 3) remove accessibility predicates and predicates that are asserted in \( \tilde{\phi} \) from \( \sigma \). Note that \( \Rightarrow \) is the consistent implication formally defined by Wise et al. [2020]. The rules for consume are given in Fig. 26.

The consume function always begins by consolidating information across the given heap \( \sigma_1.h \) and path condition \( \sigma_1.\pi \). The invariant on the heap \( \sigma_1.h \) ensures all heap chunks in \( \sigma_1.h \) are separated in memory, e.g. \( f(x; \delta_1) \in \sigma_1.h \) and \( f(y; \delta_2) \in \sigma_1.h \) implies \( x \neq y \). Similarly, \( f(x; \delta_1) \in \sigma_1.h \) implies \( x \neq \text{null} \). Therefore, such information is added to the path condition \( \sigma_1.\pi \) during consolidation. Further, consolidate ensures \( \sigma_1.h \) and \( \sigma_1.\pi \) are consistent, i.e. do not contain contradictory information. We use the definition of consolidate from [Schwerhoff 2016], without repeating it here.

After consolidation, consume calls a helper function consume', which performs the major functionality of consume. Along with the state \( \sigma_2 \) from consolidation, consume' accepts a boolean flag, optimistic heap \( \sigma_2.h_? \), regular heap \( \sigma_2.h \), the formula to be consumed \( \tilde{\phi} \), and a continuation. The boolean flag sent to consume' controls how \( \sigma_2 \) provides access to fields in \( \tilde{\phi} \). When \( \tilde{\phi} \) is precise (is \( \emptyset \)), then \( \sigma_2 \) provides access to fields in \( \emptyset \) through heap chunks or imprecision where applicable. Therefore, in this case, the boolean flag is set to \( \sigma_2.\text{isImprecise} \). However, when \( \tilde{\phi} \) is imprecise (i.e. \( ? \&\& \tilde{\phi} \)), then the boolean flag is set to true so access to fields in \( \tilde{\phi} \) is always justified: first by \( \sigma_2 \) if applicable and second by imprecision in \( \tilde{\phi} \). Copies of the optimistic heap \( \sigma_2.h_? \) and regular heap \( \sigma_2.h \) are sent to consume' where heap chunks from \( \tilde{\phi} \) are removed from them. If consume' succeeds, then when \( \tilde{\phi} \) is precise execution continues with the residual heap chunks. When \( \tilde{\phi} \) is imprecise execution continues with empty heaps, because \( \tilde{\phi} \) may require and assert any heap chunk in \( \sigma_2 \). Residual heap chunks are instead represented by imprecision, i.e. execution continues with an imprecise state. Finally, consume' also sends snapshots collected for removed heap chunks to the continuation.

Rules for consume' can also be found in Fig. 26. Cases for expressions \( e \), the separating conjunction \( \phi_1 \&\& \phi_2 \), and logical conditionals \( e \text{ ?} \phi_1 : \phi_2 \) are straightforward. Expressions are evaluated to symbolic values that are then consumed with the corresponding rule. In a separating conjunction, \( \phi_1 \) is consumed first, then afterward \( \phi_2 \) is consumed. The rule for logical conditionals evaluates the condition \( e \) to a symbolic value, and then uses the branch function to consume \( \phi_1 \) and \( \phi_2 \) along different execution paths. The case for \( \text{acc}(p(\bar{e})) \) is also very similar to the case for \( \text{acc}(e.f) \) that we discuss later in this section.

When a symbolic value \( t \) is consumed, the current state \( \sigma \) must establish \( t \), i.e. \( \sigma \Rightarrow t \), or verification fails. The assert function (defined in Fig. 28) implements this functionality. In particular, assert returns success() when \( \pi \) can statically prove \( t \) or when \( \sigma \) is imprecise and \( t \) does not contradict

\[
\begin{align*}
\text{consume}(\sigma_1, \theta, Q) &= \sigma_2 := \sigma_1 \{ h, \pi := \text{consolidate}(\sigma_1,h, \sigma_1.\pi) \} \\
\text{consume'}(\sigma_2, \sigma_2.\text{isImprecise}, \sigma_2.h_?, \sigma_2.h, \theta, (\lambda \sigma_3, h_?, h_?, h_1, \delta_1) \} \\
Q(\sigma_1 \{ h_? := h_?, h := h_1 \}, \delta_1)) \\
\text{consume}(\sigma_1, ?&\& \phi, Q) &= \sigma_2 := \sigma_1 \{ h, \pi := \text{consolidate}(\sigma_1,h, \sigma_1.\pi) \} \\
\text{consume'}(\sigma_2, \text{true}, \sigma_2.h_?, \sigma_2.h, \phi, (\lambda \sigma_3, h_?, h_1, \delta_1) \} \\
Q(\sigma_1 \{ \text{isImprecise} := \text{true}, h_? := \emptyset \}, \emptyset) \text{ pair(unit, } \delta_1) \})
\end{align*}
\]

Fig. 26. Rules for symbolically consuming formulas (1/3)
consume'($\sigma$, $f_1$, $h_1$, $(e,t)$, $Q$) = \begin{align*}
\text{res, } \overline{t} & := \text{assert}(\sigma, \text{isImprecise}, \sigma, \pi, t) \\
\mathcal{R'} & := \text{addcheck}(\sigma, \mathcal{R}, e, \text{translate}(\sigma, \overline{t})) \\
\text{res} \land Q(\sigma(\mathcal{R} := \mathcal{R'}), h_1, h, \text{unit})
\end{align*}

consume'($\sigma_1$, $f_1$, $h_1$, $h$, $e$, $Q$) = eval'($\sigma_1$, $\text{isImprecise} := f_1$, $e$, ($\lambda \sigma$, $t$.

consume'($\sigma_1$, $f_1$, $h_1$, $h$, acc($p(\overline{t})$), $Q$) = eval'($\sigma_1$, $\text{isImprecise} := f_1$, $\overline{t}$, ($\lambda \sigma$, $\overline{t}$.

consume'($\sigma_1$, $f_1$, $h_1$, $h$, acc($e.f$), $Q$) = eval'($\sigma_1$, $\text{isImprecise} := f_1$, $e$, ($\lambda \sigma$, $t$.

\begin{align*}
\text{res, } \overline{t} & := \text{assert}(\sigma, \text{isImprecise}, \sigma, \pi, t) \\
\mathcal{R}' & := \text{addcheck}(\sigma, \mathcal{R}, \text{translate}(\sigma, \overline{t})) \\
\text{res} \land Q(\sigma(\mathcal{R} := \mathcal{R'}), h_1, h, \text{null})
\end{align*}

\begin{align*}
\text{R}' & := \text{addcheck}(\sigma_1 \mathcal{R}, \text{acc}(e.f), \text{translate}(\sigma, \overline{t})) \\
(h_1, \delta_1, b_1) & := \text{heap-rem-acc}(\sigma, \text{isImprecise}, h, \sigma, \pi, f(t)) \\
\text{if } (\sigma, \text{isImprecise}) & \text{ then}
\begin{align*}
(h'_1, \delta_1, b_1) & := \text{heap-rem-acc}(\sigma, \text{isImprecise}, h, \sigma, \pi, f(t)) \\
\text{if } (b_1 = b_2 = \text{false}) & \text{ then}
\begin{align*}
\mathcal{R}' & := \text{addcheck}(\mathcal{R}', \text{acc}(e.f), \text{acc}(\text{translate}(\sigma_1, t), f)) \\
\text{else } \mathcal{R}' & := \mathcal{R}'
\end{align*}
\end{align*}
\begin{align*}
Q(\sigma(\mathcal{R} := \mathcal{R'}), h_1, h, (b_1) & \text{ then } (b_1)
\end{align*}
\end{align*}

Fig. 26. Rules for symbolically consuming formulas (2/3)

Constraints in $\pi$—here, $t$ is optimistically assumed to be true. Otherwise, assert returns failure $\bot$. When assert succeeds, it also returns a set of symbolic values $\overline{t}$ that are residuals of $t$ that cannot be proved statically by $\pi$. If $t$ is proven entirely statically, then assert returns true. A run-time check is created for the residuals $\overline{t}$ and is added to $\sigma$ to be passed to the continuation $Q$. Note that translate is used to create an expression from $\overline{t}$ that can be evaluated at run time. Further, the location $e$ is
the expression that evaluates to \( t \) and is passed to \( \text{consume'} \) alongside \( t \). The heaps \( h_2 \) and \( h \) are passed unmodified to \( Q \) alongside the snapshot \text{unit}.

The \( \text{consume'} \) rule for accessibility predicates \( \text{acc}(e.f) \), first evaluates the receiver \( e \) to \( t \) using \text{eval-c}, the current state \( \sigma_1 \), and the parameter \( f_2 \). The parameter \( f_2 \) is the boolean flag mentioned previously. Assigning \( f_2 \) to \( \sigma_1.\text{isImprecise} \) during evaluation allows \( f_2 \) to control whether or not imprecision verifies field accesses. This occurs in all of the \( \text{consume'} \) rules where expressions and thus fields are evaluated. After evaluation, the \text{isImprecise} field is reset resulting in \( \sigma_3 \), and assert is used to ensure the receiver \( t \) is non-null. If \( t \neq \text{null} \) is optimistically true, a run-time check for \( t \neq \text{null} \) at location \( \text{acc}(e.f) \) is created and added to \( \sigma_3.\mathcal{R} \). Next, heap-rem-acc is used to remove the heap chunks from heap \( h \) that overlap with or may potentially overlap with \( \text{acc}(e.f) \) in memory. The heap-rem-acc function is formally defined alongside a similar function for predicates (heap-rem-pred) in the Fig. 27. If a field chunk is not statically proven to be disjoint from \( \text{acc}(e.f) \), then it is removed. Further, since predicates are opaque, Gradual Viper cannot tell whether or not their predicate bodies overlap with \( \text{acc}(e.f) \). Therefore, predicate chunks are almost always considered to potentially overlap with \( \text{acc}(e.f) \). The only time this is not the case is if they both exist in the heap \( h \), which ensures its heap chunks do not overlap in memory. The heap-rem-acc function also checks that \( \text{acc}(e.f) \) has a corresponding heap chunk in \( h \). If so, its snapshot \( \delta_1 \) is returned and \( b_1 \) is assigned \text{true}. Otherwise, a fresh snapshot is returned with \text{false}. If the current state \( \sigma_3 \) is imprecise, then heap chunks are similarly removed from \( h_2 \) and \( \text{acc}(e.f) \) is checked for existence in \( h_7 \). If a field chunk for \( \text{acc}(e.f) \) is not found in either heap, then a run-time check is generated for it and passed to the continuation \( Q \) alongside the two heaps after removal and \( \text{acc}(e.f) \)’s snapshot. Without imprecision, \( \text{consume'} \) will fail when a field chunk for \( \text{acc}(e.f) \) is not found in \( h \).

### A.4 Symbolic execution of statements

The exec rules for sequence statements, variable declarations and assignments, allocations, and if statements are pretty much unchanged from Viper. The only difference is that Gradual Viper’s versions of \text{eval}, \text{produce}, \text{branch}, and \text{consume} (defined previously) are used instead of Viper’s. Statements in a sequence are executed one after another, and variable declarations introduce a fresh symbolic value for the variable into the state. Variable assignments evaluate the right-hand side to a symbolic value and update the variable in the symbolic store with the result. Allocations produce fresh heap chunks for fields into the state. Finally, if statements have their condition evaluated and then branch is used to split execution along two paths to symbolically execute the true and false branches.
heap-rem-pred(isImprecise, ℎ, π, ℙ(𝑡)) = if ∃ (𝐩(𝝅; 𝛿) ∈ ℎ . check(π, \{ 𝑡 = 𝑝\})) then
        (ℎ \ {𝐩(𝝅; 𝛿), 𝛿, true})
    else (∅, fresh, false)

heap-rem-acc(isImprecise, ℎ, π, 𝑓(𝑡)) = ℎ′ := foldl(ℎ, ∅, (λ 𝑠𝑟𝑐(𝝅; 𝛿). ℎ𝑑𝑠𝑡 .
                            if (¬(|𝑝(𝝅)| = 1) || ¬(𝑓 = 𝑠𝑟𝑐) || ¬check(isImprecise, π, 𝑡 = 𝑟)) then
                                ℎ𝑑𝑠𝑡 ∪ 𝑠𝑟𝑐(𝝅; 𝛿)
                            else ℎ𝑑𝑠𝑡))
    if ∃ 𝑓(𝑟; 𝛿) ∈ ℎ . check(π, 𝑡 = 𝑟) then
        (ℎ′, 𝛿, true)
    else
        ℎ′ := foldl(ℎ′, ∅, (λ 𝑠𝑟𝑐(𝝅; 𝛿). ℎ𝑑𝑠𝑡 .
                        if (𝑠𝑟𝑐(𝝅; 𝛿) is a field chunk) then
                            ℎ𝑑𝑠𝑡 ∪ 𝑠𝑟𝑐(𝝅; 𝛿)
                        else ℎ𝑑𝑠𝑡))
        (ℎ′, fresh, false)

Fig. 27. Heap remove function definitions

check(π, 𝑡) = pc-all(π) ⇒ 𝑡
check(isImprecise, π, 𝑡) =
        \{ true, true \} \text{ if } check(π, 𝑡)
        \{ true, \text{iff} (pc-all(π), 𝑡) \} \text{ if } (\text{isImprecise} \land (\bigwedge \text{pc-all(π)} \land 𝑡)_{\text{SAT}})
        \{ \text{false, 0} \} \text{ otherwise}

assert(isImprecise, π, 𝑡) =
        \{ \text{success()}, \top \} \text{ if } (𝑏 = \text{true}) \text{ where } 𝑏, \top := \text{check(isImprecise, π, 𝑡)} \text{ failure()}, \emptyset \text{ otherwise}

Fig. 28. Check and assert function definitions

Symbolic execution of field assignments first evaluates the right-hand side expression ℒ to the
symbolic value 𝑡 with the current state σ₁ and eval. Any field reads in ℒ are either directly
or optimistically verified using σ₁. Then, the resulting state σ₂ must establish write access to 𝑥.𝑓 in
consume, i.e. σ₂ ⊨ acc(𝑥.𝑓). The call to consume also removes the field chunk for acc(𝑥.𝑓) from
σ₂ (if it is in there) resulting in σ₃. Therefore, the call to produce can safely add a fresh field chunk
for acc(𝑥.𝑓) alongside 𝑥.𝑓 = 𝑡 to σ₃ before it is passed to the continuation Q. Under the hood,
run-time checks are collected where required for soundness and passed to Q.

The exec rule for method calls similarly uses eval to evaluate the given args 𝔇 to symbolic values
意义 𝔇, asserts the method’s precondition methpre holds in the current state, consumes the heap chunks
in the preconditions, and produces the method’s postcondition methpost into the continuation. Run-
time checks are also collected where necessary (under the hood) and passed to the continuation.
Gradual Viper makes an exception when consuming preconditions at method calls (and loop
invariants before entering loops), which can be seen in the if-then in the method call rule. If
Gradual Viper determines the precondition (invariant) is equi-recursively imprecise, then it will
conservatively remove all the heap chunks from both symbolic heaps after the consume. This
exception ensures the static verification semantics in Gradual Viper lines up with the equi-recursive,
dynamic verification semantics encoded by GVC0 in §4.4 such that Gradual C₀ is sound. Note that
the origin field of 𝑅 is set to 𝔇 := m(𝔻) before consuming methpre and reset to none after producing
methpost. Setting the origin indicates that run-time checks or branch conditions for methpre or
\begin{verbatim}
exec(\sigma_1, s_1; s_2, Q) = exec(\sigma_1, s_1, (\lambda\sigma_2 . exec(\sigma_2, s_2, Q)))
exec(\sigma, \text{var } x : T, Q) = Q(\sigma[y := \text{havoc}(\sigma.y, x)])
exec(\sigma_1, x := e, Q) = \text{eval}(\sigma_1, e, (\lambda\sigma_2, t . Q(\sigma_2[y := \sigma_2.y[x \mapsto t]])))
exec(\sigma_1, x.f := e, Q) = \text{eval}(\sigma_1, e, (\lambda\sigma_2, t : \text{consume}(\sigma_2, acc(x.f), (\lambda\sigma_3, _ . produe(\sigma_3, acc(x.f) & & x.f = t, \text{pair}\text{(fresh, unit)}, Q)))))
exec(\sigma, x := \text{new}(\tilde{f}), Q) = \text{produce}(\sigma[y := \text{havoc}(\sigma.y, x)], \tilde{acc}(x.f), \text{fresh}, Q)
exec(\sigma_1, \text{\_} := m(\bar{\tau}), Q) = \text{eval}(\sigma_1, \bar{\tau}, (\lambda\sigma_2, \tilde{\bar{\tau}}).
\begin{align*}
R' &= \sigma_2.R[\text{origin} := (\sigma_2, \bar{\tau} := m(\bar{\tau}), \bar{T})] \\
\text{consume}(\sigma_2[R := R'], \text{meth}_{\text{pre}}[\text{meth}_{\text{args}} \mapsto \bar{T}], (\lambda\sigma_3, \delta \ . \text{if (equi-imp(meth_{\text{pre}})) then})
\sigma_4 := \sigma_3\{\text{isImprecise} := \text{true}, h_1 := \emptyset, h := \emptyset,
\bar{y} := \text{havoc}(\bar{\sigma}_3.y, \bar{\tau})\}
\text{else } \sigma_4 := \sigma_3\{\bar{y} := \text{havoc}(\bar{\sigma}_3.y, \bar{\tau})\}
\text{produce}(\sigma_4, \text{meth}_{\text{post}}[\text{meth}_{\text{args}} \mapsto \bar{T}][\text{meth}_{\text{ex}} \mapsto \bar{z}], \text{fresh}, (\lambda\sigma_5 . Q(\sigma_5[R := \sigma_3.R[\text{origin} := \text{none}]])))
\end{align*}
\end{verbatim}

\text{exec}(\sigma_1, \text{assert } \phi, Q) = \text{consume}(\sigma_1, \phi, (\lambda\sigma_2, \delta \ . \\
\text{well-formed}(\sigma_2, ? & & \phi, \delta, (\lambda\sigma_3 . \\
Q(\sigma_3[\bar{\pi} := \sigma_3.\pi, R := \sigma_3.R]))))
\text{exec}(\sigma_1, \text{fold acc}(p(\bar{\tau})), Q) = \text{eval}(\sigma_1, \bar{\tau}, (\lambda\sigma_2, \tilde{\bar{\tau}}).
\begin{align*}
R' &= \sigma_2.R[\text{origin} := (\sigma_2, \text{fold acc}(p(\bar{\tau})), \bar{T})]
\text{consume}(\sigma_2[R := R'], \text{pred}_{\text{body}}[\text{pred}_{\text{args}} \mapsto \bar{T}], (\lambda\sigma_3, \delta \ . \\
\text{produce}(\sigma_3[R := \sigma_3.R[\text{origin} := \text{none}]], \tilde{acc}(p(\bar{T})), \delta, (Q)))
\end{align*}
\text{exec}(\sigma_1, \text{unfold acc}(p(\bar{\tau})), Q) = \text{eval}(\sigma_1, \bar{\tau}, (\lambda\sigma_2, \tilde{\bar{\tau}}).
\begin{align*}
R' &= \sigma_2.R[\text{origin} := (\sigma_2, \text{unfold acc}(p(\bar{\tau})), \bar{T})]
\text{consume}(\sigma_2[R := R'], \text{acc}(p(\bar{T})), (\lambda\sigma_3, \delta \ . \\
\text{produce}(\sigma_3[\text{pred}_{\text{body}}[\text{pred}_{\text{args}} \mapsto \bar{T}], \delta, (\lambda\sigma_4 . \text{Q}(\sigma_4[R := \sigma_3.R[\text{origin} := \text{none}]]))))
\end{align*}
\text{exec}(\sigma_1, \text{if } (e) \{ \text{stmt}_1 \} \text{ else } \{ \text{stmt}_2 \}, Q) = \text{eval}(\sigma_1, e, (\lambda\sigma_2, t . \\
\text{branch}(\sigma_2, e, t, (\lambda\sigma_3 . \text{exec}(\sigma_3, \text{stmt}_1, Q)), (\lambda\sigma_4 . \text{exec}(\sigma_4, \text{stmt}_2, Q))))
\end{verbatim}

Fig. 29. Rules for symbolically executing program statements (1/2)

\text{meth}_{\text{post}} should be attached to the method call statement rather than where they are declared. The origin arguments \sigma_2 and \tilde{\bar{t}} are used to reverse the substitution \text{[meth}_{\text{args}} \mapsto \bar{T}] in run-time checks and branch conditions for \text{meth}_{\text{pre}} and \text{meth}_{\text{post}}. The rule for (un)foling predicates operates the same as for method calls where \text{meth}_{\text{pre}} is the predicate body (predicate instance) and \text{meth}_{\text{post}} is the predicate instance (predicate body). The origin is set to fold \tilde{\text{acc}}(p(\bar{\tau})) and unfold \text{acc}(p(\bar{\tau})) respectively.
exec(σ₁, while (e invariant φ) { stmt }, Q) = γ₁ := havoc(σ₁, x, R)

\[ res_{body} := \text{well-formed (} \]
\[ σ₁\{ isImprecise := false, h₁ := 0, h := 0, y := γ₁, \]
\[ R := σ₁,R\{origin := (σ₁, while (e) invariant φ) { stmt }, beginning}\}, \]
\[ φ \&\& e, fresh, (λ \ σ₂\{ R := σ₁,R\{origin := none}\} . \]
\[ exec(σ₂, stmt, (λ \ σ₄) . \]
\[ eval (σ₄\{ R := σ₁,R\{origin := (σ₄, while (e) invariant φ) { stmt }, end}\}, \]
\[ e, (λ \ σ₅) . \]
\[ consume (σ₅\{ R := σ₁,R\{origin := (σ₅, while (e) invariant φ) { stmt }, before}\}, \]
\[ e, (λ \ σ₆) . \]
\[ res_{after} := eval (σ₁\{ R := σ₁,R\{origin := (σ₁, while (e) invariant φ) { stmt }, after}\}, \]
\[ if (\text{equi-imp}(φ)) \]
\[ σ₃ := σ₁\{ isImprecise := true, h₁ := 0, h := 0, y := γ₂\} \]
\[ else σ₃ := σ₁\{ y := γ₂\} \]
\[ produce (σ₃\{ R\{origin := (σ₃, while (e) invariant φ) { stmt }, after}\}, \]
\[ φ \&\& e, fresh, (Q))))) \]
\[ if (σ₁.isImprecise) \]
\[ if (¬res_{body} ∧ res_{after}) \]
\[ R′ := addcheck(σ₁,R, e, ¬e) \]
\[ R := R ∪ R′.rcs.last \]
\[ (¬res_{body} ∨ res_{after}) ∧ (res_{body} ∨ res_{after}) \]
\[ else res_{body} ∧ res_{after} \]

where x are variables modified by the loop body.

Fig. 29. Rules for symbolically executing program statements (2/2)

In contrast, φ in assert φ maintains a none origin field, because φ’s use and declaration align at the same program location assert φ. The assert rule relies on consume to assert φ holds in the current state σ₁. If the consume succeeds, the state σ₁ is passed to the continuation nearly unmodified. Path condition constraints from φ hold in σ₁ either directly or optimistically. Therefore, these constraints are added to σ₁ to avoid producing run-time checks for them in later program statements. Run-time checks from the consume are also passed to the continuation. Note that φ is checked for well-formedness here (Fig. 31). A formula is well-formed if it contains ? or accessibility predicates that verify access to the formula’s fields (self-framing). Additionally, the formula cannot contain duplicate accessibility predicates or predicate instances. Finally, well-formed adds the
\[
equi-\text{imp}(\land \land \phi) = \text{true}
\]
\[
equi-\text{imp}(\phi_1 \land \land \phi_2) = \text{equi-imp}(\phi_1) \lor \text{equi-imp}(\phi_2)
\]
\[
equi-\text{imp}(e \ ? \phi_1 : \phi_2) = \text{equi-imp}(\phi_1) \lor \text{equi-imp}(\phi_2)
\]
\[
equi-\text{imp}(\text{acc}(\rho(\overline{r}))) = \text{if} \ (p \in \text{VisitedPreds}) \ \text{then}
\]
\[
\text{false}
\]
\[
\text{else}
\]
\[
\text{VisitedPreds} := \text{VisitedPreds} \cup p
\]
\[
b := \text{equi-imp}(\text{pred}_{\text{body}})
\]
\[
\text{VisitedPreds} := \text{VisitedPreds} \setminus p
\]
\[
b
\]
\[
equi-\text{imp}(\_ ) = \text{false}
\]

Fig. 30. Boolean function determining if a gradual formula is equi-recursively imprecise or not

formula’s information to the given symbolic state. Here, \(\phi\) does not need to be self-framed, and so it is joined with \(?\) in the call to well-formed. \(?\) verifies access to all of \(\phi\)’s fields.

\[
\text{well-formed}(\sigma_1, \overline{\phi}, \delta, Q) = \text{produce}(\sigma_1, \overline{\phi}, \delta, (\lambda \sigma_2 \ .
\]
\[
\text{produce}(\sigma_1 \{ \pi := \sigma_2.\pi \}, \overline{\phi}, \delta, Q)))
\]

Fig. 31. Well-formed formula function definition

Finally, while the while loop rule is the largest rule and looks fairly complex, it just combines ideas from other rules that are discussed in great detail in this section and from the branch rule described in §A.2.

A.5 Valid program

A Gradual Viper program is valid if all of its method and predicate declarations are verified successfully as defined in Fig. 15. In particular, a method \(m\)’s declaration is verified first by checking well-formedness of \(m\)’s precondition \(\text{meth}_{\text{pre}}\) and postcondition \(\text{meth}_{\text{post}}\) using the empty state \(\sigma_0\) (well-formedness is described in §A.4). Note, fresh symbolic values are created and added to \(\sigma_0\) for \(m\)’s argument variables \(\overline{x}\) and return variables \(\overline{y}\). If \(\text{meth}_{\text{pre}}\) and \(\text{meth}_{\text{post}}\) are well-formed, then the body of \(m\) (\(\text{meth}_{\text{body}}\)) is symbolically executed (§A.4) starting with the symbolic state \(\sigma_1\) containing \(\text{meth}_{\text{pre}}\). Recall, well-formed additionally produces the formula that is being checked into the symbolic state. The symbolic state \(\sigma_2\) is produced after the symbolic execution of \(\text{meth}_{\text{body}}\). Then, \(\text{meth}_{\text{post}}\) is checked for validity against \(\sigma_2\), i.e. \(\sigma_2\) must establish \(\text{meth}_{\text{post}}\) (§A.3). If \(\text{meth}_{\text{post}}\) is established, then verification succeeds; and as a result, the run-time checks collected during verification are added to \(\mathcal{R}\) (highlighted in blue). A valid predicate \(p\) is simply valid if \(p\)’s body \(\text{pred}_{\text{body}}\) is well-formed. As before, fresh symbolic values are created for \(p\)’s argument variables \(\overline{x}\). Note, no run-time checks are added to \(\mathcal{R}\) here, because well-formedness checks do not produce any run-time checks.